Note

- Conjugate binomials are pairs of binomials in the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$.

- The product of conjugate binomials is always a rational number of the form $a^2b - c^2d$.

Class Ex. #4   Write the conjugate of each. Then multiply each pair.

a) $4\sqrt{6} + 3$ b) $-3\sqrt{11} + \sqrt{2}$ c) $5\sqrt{x} - \sqrt{y}$

$$3$$
$$16$$
$$\underline{5}$$
$$0$$

60

Complete Assignment Questions #11 - #17

Assignment

1. Multiply and simplify where possible. Do not use a calculator.

a) $(\sqrt{7})(\sqrt{3})$

$\sqrt{21}$

b) $4\sqrt{3} \times 2\sqrt{5}$

$8\sqrt{15}$

c) $-3\sqrt{5} \times 2\sqrt{2}$

$-6\sqrt{10}$

d) $6\sqrt{p} \times 8\sqrt{q}$

$48\sqrt{pq}$

e) $(\sqrt{15})(\sqrt{3})$

$\sqrt{45}$
$9 \cdot 5$
$3\sqrt{5}$

f) $10\sqrt{5} \times 9\sqrt{5}$

$90\sqrt{25}$
$90(5)$
450

g) $3\sqrt{6} \cdot 5\sqrt{10}$

$15\sqrt{60}$
$4 \cdot 15$
$15 \cdot 2\sqrt{15}$
$30\sqrt{15}$

h) $\sqrt{a} \times 10\sqrt{a}$

$10\sqrt{a^2}$
$10a$

i) $7\sqrt{54} \cdot 2\sqrt{6}$

$14\sqrt{324}$
$14(18)$
252

j) $(\sqrt{32})(\sqrt{6})$

$\sqrt{192}$
$64 \cdot 3$
$8\sqrt{3}$

k) $\sqrt{15} \times 3\sqrt{27}$

$3\sqrt{405}$
$81 \cdot 5$
$3(9)\sqrt{5}$
$27\sqrt{5}$

2. In each case, write each radical as the product of two mixed radicals in two different ways.

a) $15\sqrt{18}$ $15: 3\times 5$

$3\sqrt{2} \cdot 5\sqrt{9}$ $18: 2\times 9$

$3\sqrt{3} \circ 5\sqrt{6}$ $3 \cdot 6$

b) $35\sqrt{6}$ $35: 7\times 5$ $6: 2\times 3 , 1\times 6$

$7\sqrt{2} \cdot 5\sqrt{3}$

$7\sqrt{1} \cdot 5\sqrt{6}$

3. Express in simplest form. Do not use a calculator.

a) $\left(\sqrt{3}\right)^2$

$(\sqrt{3})(\sqrt{3})$

$\sqrt{9}$

3

b) $\left(4\sqrt{2}\right)^2$

$(4\sqrt{2})(4\sqrt{2})$

$16\sqrt{4}$

$16(2) = 32$

c) $\left(-3\sqrt{5}\right)^2$

$(-3\sqrt{5})(-3\sqrt{5})$

$9(5)$

45

d) $-\left(\sqrt{12}\right)^2$

$-(\sqrt{12})(\sqrt{12})$

-12

e) $\left(\sqrt{5}\right)^3$

$(\sqrt{5})(\sqrt{5})(\sqrt{5})$

$\sqrt{125}$ (25.5)

$5\sqrt{5}$

4. Express in simplest form.

a) $\sqrt{5} \times 2\sqrt{3} \times 3\sqrt{2}$

$6\sqrt{30}$

b) $2\sqrt{6} \times 2\sqrt{3} \times 3\sqrt{2}$

$12\sqrt{36}$

$12(6)$

72

c) $\left(-2\sqrt{6}\right)\left(2\sqrt{3}\right)\left(-3\sqrt{5}\right)$

$\cdot 12\sqrt{90}$

$9 \cdot 10$

$12 \cdot 3\sqrt{10}$

$36\sqrt{10}$

d) $\left(\frac{2}{3}\sqrt{27}\right)\left(\sqrt{6}\right)$

$\frac{2}{3}\sqrt{162}$

$81 \cdot 2$

$\frac{2}{3}(9)\sqrt{2}$

$6\sqrt{2}$

e) $2\sqrt{\dfrac{8}{25}} \times 5\sqrt{2}$

$10\sqrt{\dfrac{16}{25}}$

$10\dfrac{\sqrt{16}}{\sqrt{25}}$

$10\left(\dfrac{4}{5}\right)$

8

f) $3\sqrt[3]{16} \times 2\sqrt[3]{4} \times 2\sqrt[3]{2}$

$12\sqrt[3]{128}$

$64 \cdot 2$

$12(4)\sqrt[3]{2}$

$48\sqrt[3]{2}$

5. Consider the product $6\sqrt{5} \times 3\sqrt{8}$.

a) Use a **two decimal place approximation** for each radical to determine a two decimal place approximation for the product.

b) Determine the **exact value** of the product as a mixed radical in simplest form.

c) Determine a **two decimal place approximation** to the answer in b).

d) Which of the two decimal place approximations is more accurate? Explain.

6. Expand and simplify where possible.

a) $\sqrt{6}\left(2\sqrt{6} - \sqrt{5}\right)$

$2(6) - \sqrt{30}$

$12 - \sqrt{30}$

b) $\sqrt{2}\left(1 - \sqrt{2}\right)$

$\sqrt{2} - 2$

c) $2\sqrt{3}\left(2\sqrt{7} - 4\sqrt{5}\right)$

$4\sqrt{21} - 8\sqrt{15}$

7. Expand and simplify.

a) $\sqrt{3}\left(2\sqrt{6} - \sqrt{12}\right)$

$2\sqrt{18} - \sqrt{36}$
$\quad\;\; 9 \cdot 2$

$2(3)\sqrt{2} - 6$

$6\sqrt{2} - 6$

b) $\sqrt{8}\left(\sqrt{6} - \sqrt{2}\right)$

$\sqrt{48} - \sqrt{16}$
$\;\;16 \cdot 3$

$4\sqrt{3} - 4$

c) $\sqrt{y}\left(\sqrt{x} - 9\sqrt{y}\right)$

$\sqrt{xy} - 9y$

d) $2\sqrt{11}\left(3\sqrt{2} - \sqrt{50} + 3\sqrt{32}\right)$

$6\sqrt{22} - 2\sqrt{550} + 6\sqrt{352}$
$\qquad\quad 25 \cdot 22 \qquad 16 \cdot 22$

$6\sqrt{22} - 2(5)\sqrt{22} + 6(4)\sqrt{22}$

$6\sqrt{22} - 10\sqrt{22} + 24\sqrt{22}$

$20\sqrt{22}$

e) $\sqrt{5}\left(3\sqrt{5} - \sqrt{75} + 3\sqrt{3}\right)$

$3\sqrt{25} - \sqrt{375} + 3\sqrt{15}$
$\qquad\qquad 25 \cdot 15$

$3(5) - 5\sqrt{15} + 3\sqrt{15}$

$15 - 2\sqrt{15}$

8. Simplify. FOIL or GRID

a) $\left(4 + \sqrt{27}\right)\left(1 - \sqrt{12}\right)$

$4 - 4\sqrt{12} + \sqrt{27} - \sqrt{324}$
$\qquad\;\; 4 \cdot 3 \qquad 9 \cdot 3$

$4 - 4(2)\sqrt{3} + 3\sqrt{3} - 18$

$-14 - 8\sqrt{3} + 3\sqrt{3}$

$-14 - 5\sqrt{3}$

b) $\left(2\sqrt{3} - \sqrt{10}\right)\left(\sqrt{6} - 7\sqrt{20}\right)$

	$2\sqrt{3}$	$-\sqrt{10}$
$\sqrt{6}$	$2\sqrt{18}$	$-\sqrt{60}$
	$9 \cdot 2$	$4 \cdot 15$
$-7\sqrt{20}$	$-14\sqrt{60}$	$+7\sqrt{200}$
	$4 \cdot 15$	$100 \cdot 2$

$6\sqrt{2} - 2\sqrt{15} - 28\sqrt{15} + 70\sqrt{2}$

$76\sqrt{2} - 30\sqrt{15}$

9. Write and simplify an expression for the area of each shape.

a)

$5 + \sqrt{3}$

$5 - \sqrt{3}$

$(5-\sqrt{3})(5+\sqrt{3})$

$25 + 5\sqrt{3} - 5\sqrt{3} - 3$

22

b)

$\sqrt{2} + \sqrt{3}$
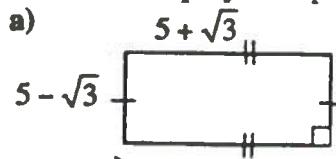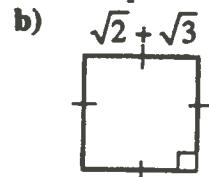
$(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})$

$2 + \sqrt{6} + \sqrt{6} + 3$

$5 + 2\sqrt{6}$

c) rectangle $2\sqrt{10}$ by $\left(\sqrt{6}+4\sqrt{5}\right)$

$2\sqrt{10}(\sqrt{6}+4\sqrt{5})$

$2\sqrt{60} + 8\sqrt{50}$
  4·15   25·2

$2(2)\sqrt{15} + 8(5)\sqrt{2}$

$4\sqrt{15} + 40\sqrt{2}$

d) square with sides $3\sqrt{208} - 8$

$(3\sqrt{208}-8)(3\sqrt{208}-8)$

* Sometimes its better to simplify first *

$3\sqrt{208}$
16·13
$3(4)\sqrt{13}$

$\rightarrow (12\sqrt{13}-8)(12\sqrt{13}-8)$

$144(13) - 96\sqrt{13} - 96\sqrt{13} + 64$

$1872 - 192\sqrt{13} + 64$

$1936 - 192\sqrt{13}$

10. Expand and simplify.

a) $\left(5\sqrt{3}-2\right)^2$

$(5\sqrt{3}-2)(5\sqrt{3}-2)$

$25(3) - 10\sqrt{3} - 10\sqrt{3} + 4$

$79 - 20\sqrt{3}$

b) $\left(4\sqrt{6}-\sqrt{2}\right)^2$

$(4\sqrt{6}-\sqrt{2})(4\sqrt{6}-\sqrt{2})$

$16(6) - 8\sqrt{6} - 8\sqrt{6} + 2$

$98 - 16\sqrt{6}$

c) $2\left(\sqrt{15}-3\sqrt{5}\right)^2$

$3\sqrt{75}$
25·3
$3(5)\sqrt{3}$
$15\sqrt{3}$

$2\left[(\sqrt{15}-3\sqrt{5})(\sqrt{15}-3\sqrt{5})\right]$

$2\left(15 - 3\sqrt{75} - 3\sqrt{75} + 9(5)\right]$
        25·3   25·3

$2\left[60 - 30\sqrt{3}\right]$

$120 - 60\sqrt{3}$

d) $\left(7\sqrt{x}-2\sqrt{y}\right)^2$

$(7\sqrt{x}-2\sqrt{y})(7\sqrt{x}-2\sqrt{y})$

$49x - 14\sqrt{xy} - 14\sqrt{xy} + 4y$

$49x + 4y - 28\sqrt{xy}$

these are conjugates middle term eliminates.

11. Expand and simplify.

a) $(\sqrt{5}+1)(\sqrt{5}-1)$ b) $(\sqrt{8}+\sqrt{7})(\sqrt{8}-\sqrt{7})$ c) $(2\sqrt{6}-\sqrt{2})(2\sqrt{6}+\sqrt{2})$

$5-\sqrt{5}+\sqrt{5}-1$ $8-7$ $4(6)-2$

4 1 22

12. Write the conjugate of each.

a) $\sqrt{2}-\sqrt{5}$ b) $4+\sqrt{7}$ c) $-3\sqrt{8}-15$

$\sqrt{2}+\sqrt{5}$ $4-\sqrt{7}$ $-3\sqrt{8}+15$

13. Write the conjugate of each. Then multiply each pair.

a) $(\sqrt{3}-1)(\sqrt{3}+1)$ b) $(2+\sqrt{5})(2-\sqrt{5})$ c) $(2\sqrt{6}-\sqrt{3})(2\sqrt{6}+\sqrt{3})$

$3-1$ $4-5$ $4(6)-3$

2 -1 21

d) $(2\sqrt{8}+\sqrt{27})(2\sqrt{8}-\sqrt{27})$ e) $(\sqrt{32}-\sqrt{3})(\sqrt{32}+\sqrt{3})$ f) $(-3\sqrt{40}+2\sqrt{10})(-3\sqrt{40}-2\sqrt{10})$

$4(8)-27$ $32-3$ $9(40)-4(10)$

5 29 $360-40$

320

14. For all values of a and b, $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$ is equal to

A. $\sqrt{(a-b)(a+b)}$

B. $a-b$

C. $a+b$

D. a^2-b^2

$a-\sqrt{ab}+\sqrt{ab}-b$

15. $(\sqrt{2})^5$ is equal to

A. $\sqrt{10}$
B. $5\sqrt{2}$
C. $4\sqrt{2}$
D. 32

$(\sqrt{2})(\sqrt{2})(\sqrt{2})(\sqrt{2})(\sqrt{2})$

$(2)(2)\sqrt{2}$

$4\sqrt{2}$

16. The expression $\sqrt{5}\left(\sqrt{10} + 12\sqrt{5}\right) - \sqrt{7}\left(\sqrt{7} - 2\sqrt{14}\right)$ can be simplified to the form $a + b\sqrt{c}$ where a, b and c are integers. The value of $a + b + c$ is _____.

(Record your answer in the numerical response box from left to right.)

[handwritten work]

$\sqrt{5}\left(\sqrt{10} + 12\sqrt{5}\right) - \sqrt{7}\left(\sqrt{7} - 2\sqrt{14}\right)$

$\sqrt{50} + 12(5) - \left(7 - 2\sqrt{98}\right)$

$25 \cdot 2 \qquad\qquad 49 \cdot 2$

$5\sqrt{2} + 60 - 7 + 14\sqrt{2}$

$53 + 19\sqrt{2}$
$\ \ \ a \quad\ b\ \ \ c$

$53 + 19 + 2 = 74$

17. If $p \oplus q$ means "$(p - q)$ multiplied by q" then the value of $\sqrt{6} \oplus \sqrt{3}$ can be simplified to the form $a + b\sqrt{c}$ where a, b and c are integers. $p \quad q$
The value of c is _____.

(Record your answer in the numerical response box from left to right.)

[handwritten work]

$\sqrt{3}\left(\sqrt{6} - \sqrt{3}\right)$

$\sqrt{18} - 3$
$9 \cdot 2$

$3\sqrt{2} - 3 \quad\rightarrow\quad -3 + 3\sqrt{2}$
$\qquad\qquad\qquad\qquad a \quad\ b \quad c$

Answer Key

1. a) $\sqrt{21}$ b) $8\sqrt{15}$ c) $-6\sqrt{10}$ d) $48\sqrt{pq}$ e) $3\sqrt{5}$ f) 450 g) $30\sqrt{15}$
   h) $10a$ i) 252 j) $8\sqrt{3}$ k) $27\sqrt{5}$

2. Answers may vary.
   a) $(3\sqrt{3})(5\sqrt{6})$ or $(5\sqrt{3})(3\sqrt{6})$ b) $(5\sqrt{2})(7\sqrt{3})$ or $(7\sqrt{2})(5\sqrt{3})$

3. a) 3 b) 32 c) 45 d) -12 e) $5\sqrt{5}$

4. a) $6\sqrt{30}$ b) 72 c) $36\sqrt{10}$ d) $6\sqrt{2}$ e) 8 f) $48\sqrt[3]{2}$

5. a) 113.94 b) $36\sqrt{10}$ c) 113.84 d) c) because rounding is not done until the last step.

6. a) $12 - \sqrt{30}$ b) $\sqrt{2} - 2$ c) $4\sqrt{21} - 8\sqrt{15}$

7. a) $6\sqrt{2} - 6$ b) $4\sqrt{3} - 4$ c) $\sqrt{xy} - 9y$ d) $20\sqrt{22}$ e) $15 - 2\sqrt{15}$

8. a) $-14 - 5\sqrt{3}$ b) $76\sqrt{2} - 30\sqrt{15}$

9. a) 22 b) $5 + 2\sqrt{6}$ c) $4\sqrt{15} + 40\sqrt{2}$ d) $1936 - 192\sqrt{13}$

10. a) $79 - 20\sqrt{3}$ b) $98 - 16\sqrt{3}$ c) $120 - 60\sqrt{3}$ d) $49x - 28\sqrt{xy} + 4y$

11. a) 4 b) 1 c) 22

12. a) $\sqrt{2} + \sqrt{5}$ b) $4 - \sqrt{7}$ c) $-3\sqrt{8} + 15$

13. a) $\sqrt{3} + 1, 2$ b) $2 - \sqrt{5}, -1$ c) $2\sqrt{6} + \sqrt{3}, 21$
    d) $2\sqrt{8} - \sqrt{27}, 5$ e) $\sqrt{32} + \sqrt{3}, 29$ f) $-3\sqrt{40} - 2\sqrt{10}, 320$

14. B   15. C   16. | 7 | 4 | | | 17. | 2 | | | |