Analyzing Radical and Rational Functions Lesson #7: Practice Test

Section A *No calculator may be used for this section of the test.*

1. The domain and range of the function $y = -\sqrt{-x+1}$ are, respectively, all real numbers such that

   A. $x \geq 1, y \geq 0$
   B. $x \geq -1, y \leq 0$
   C. $x \leq -1, y \geq 0$
   D. $x \leq 1, y \leq 0$

$-x+1 \geq 0$
$1 \geq x$
$x \leq 1 \quad y \leq 0$

Use the following information to answer the next two questions.

> A student is analyzing the graph of the function $y = g(x)$. She correctly deduces that the range of the function $y = \sqrt{g(x)}$ is $\{y \mid 0 \leq y \leq 4, y \in R\}$.
>
> She makes four statements about the graph of $y = g(x)$.
>
> **Statement 1:** The point $(1, 2)$ lies on the graph of $y = g(x)$.
> **Statement 2:** The graph of $y = g(x)$ has no x-intercepts.
> **Statement 3:** The graph of $y = g(x)$ has no points in quadrants three or four.
> **Statement 4:** The maximum value of $y = g(x)$ is 2.

2. Which statement(s) **must** be false?

   A. Statement 2 only
   B. Statement 4 only
   C. Statement 2 and 4 only
   D. Some other statement, or combination of statements, must be false.

Statement 1/3 may or may not be false
Statement 2 : $\sqrt{g(x)}$ has an x-int so $g(x)$ has an x-int
Statement 4 : max value of $g(x)$ is $4^2 = 16$

3. Which statement(s) **must** be true?

   A. Statement 1 only
   B. Statement 3 only
   C. None of the statements must be true.
   D. Some other statement, or combination of statements, must be true.

Use the following information to answer the next three questions.

Consider the graph of the rational function $f(x) = \dfrac{x-7}{x^2 - 9x + 14}$.

4. Which one of the following statements is true regarding the graph of $f(x)$?

A. The horizontal asymptote has equation $y = 0$.
B. The horizontal asymptote has equation $y = 1$.
C. The horizontal asymptote has equation $x = 7$.
D. There is no horizontal asymptote.

$f(x) = \dfrac{x-7}{(x-7)(x-2)} = \dfrac{1}{(x-2)}, x \neq 7$

degree of numerator < degree of denominato

5. Which one of the following statements is true regarding the graph of $f(x)$?

A. The vertical asymptote has equation $x = 0$.
B. The vertical asymptote has equation $x = 2$.
C. The vertical asymptote has equation $x = 7$.
D. There is no vertical asymptote.

Numerical Response 1. If the point of discontinuity of the graph can be represented by (a, b), then the value of $a + b$, correct to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right.)

7	.	2	

replace 7 in $\dfrac{1}{x-2}$ $\dfrac{1}{7-2}$ $\to \dfrac{1}{5}$ $a = 7$ $b = \dfrac{1}{5}$

$a+b = 7.2$

Section B ***A graphing calculator may be used for the remainder of the test.***

6. The range of $y = \sqrt{f(x)}$ is $\{y \mid 3 \le y \le 12, y \in R\}$. Which one of the following points could lie on the graph of $y = f(x)$?

A. $(3, \sqrt{3})$ B. $(9, 2)$
C. $(8, 5)$ D. $(18, 18)$

In Q 1, range of $y = f(x)$ is $9 \le y \le 144$.

7. Which of the following statements concerning the roots of the equation $\dfrac{2x^3 - 19x^2 + 56x - 48}{x - 4} = 0$ is correct?

A. The only root is 1.5.
B. The only root is 4.0.
C. There are two roots, 1.5 and 4.0.
D. There are no roots.

4 in non permissible

$$\begin{array}{r|rrrr} 4 & 2 & -19 & 56 & -48 \\ & & 8 & -44 & 48 \\ \hline & 2 & -11 & 12 & 0 \end{array}$$

$= (x-4)(2x^2 - 11x + 12)$

$= (x-4)(x-4)(2x-3)$

root $= 3/2$

Numerical Response **2.** To the nearest hundredth, the solution of the equation $\frac{x-9}{8-x} = 20$ is _____.

(Record your answer in the numerical response box from left to right.)

8	.	0	5

$x - 9 = 20(8-x)$

$x - 9 = 160 - 20x$

$21x = 169$

$x = \frac{169}{21} = 8.047$

Use the following information to answer the next five questions.

Consider the function $f(x) = 2 - 4\sqrt{5x-3}$.

8. The range of the function $y = f(x)$ is

A. $\{y \mid y \in R\}$ **B.** $\{y \mid -2 \le y \le 2, y \in R\}$

C. $\{y \mid y \le 2, y \in R\}$ **D.** $\{y \mid y \ge 2, y \in R\}$

max value of $f = 2 - 4(0) =$

no min value.

Numerical Response **3.** The domain of the function $y = f(x)$ is $x \ge c$. The value of c, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right.)

$5x - 3 \ge 0$, $5x \ge 3$ $x \ge 3/5$

0	.	6	

9. If $g(x) = \sqrt{f(x)}$, then the domain of $g(x)$ is

A. $\{x \mid x \ge 0, x \in R\}$

B. $\{x \mid x \ge 0.6, x \in R\}$

C. $\{x \mid x \ge 0.65, x \in R\}$

D. $\{x \mid 0.6 \le x \le 0.65, x \in R\}$

$f(x) = 0 \Rightarrow 2 - 4\sqrt{5x-3} = 0$

$x = 0.65$

For $x > 0.65$ $f(x) < 0$ and $g(x)$ does not exist

$f(x) \ge 0$ on $0.6 \le x \le 0.65$

10. The solution of the equation $f(x) = -5$ can be determined from the x-intercepts of the graph of

A. $y = 2 - 4\sqrt{5x-3}$

B. $y = -3 - 4\sqrt{5x-3}$

C. $y = -7 - 4\sqrt{5x-3}$

D. $y = 7 - 4\sqrt{5x-3}$

$-5 = 2 - 4\sqrt{5x-3}$

$0 = 7 - 4\sqrt{5x-3}$

Numerical Response **4.** The solution of the equation $f(x) = -5$, to the nearest hundredth, is _____.

(Record your answer in the numerical response box from left to right.)

1	.	2	1

$7 = 4\sqrt{5x-3}$

$\frac{7}{4} = \sqrt{5x-3}$

$\frac{49}{16} = 5x - 3$

$\frac{97}{16} = 5x$

$x = \frac{97}{80} = 1.2125$

Use the following information to answer the next two questions.

> $y = f(x)$ is a continuous function with zeros -5, 1, and 4, and domain $x \in R$
> The domain of $y = \sqrt{f(x)}$ is the set of real numbers such that $x \leq -5$ or $x \geq 4$.
>
> Zach has been given a code to describe whether the function is positive, negative, zero, or does not exist for particular values of x.
>
> For the function value $f(x_0)$, he used the following code:
> - If the function is **negative** at $x = x_0$, use the code number 1.
> - If the function is **zero** at $x = x_0$, use the code number 2.
> - If the function is **positive** at $x = x_0$, use the code number 3.
> - If the function **does not exist** at $x = x_0$, use the code number 4.

Numerical Response **5.** In the first box, write the code number for $f(-6)$.
In the second box, write the code for $f(-3)$.
In the third box, write the code number for $f(1)$.
In the last box, write the code number for $f(6)$.

(Record your answer in the numerical response box from left to right.)

3	1	2	3

11. If the function $f(x)$ has the lowest possible degree, then the multiplicity of the zero at 1 is

A. 1 **B.** 2 **C.** 3
D. unable to be determined

Use the following information to answer the next three questions.

> Consider the functions
>
> $$f(x) = \frac{x+a}{x^2+a},\quad g(x) = \frac{(x+a)(x+c)}{x^2+xa+xb+ab},\quad h(x) = \frac{2x+b}{x^2-b},\quad k(x) = \frac{x^2+b}{x^2+a}$$
>
> where a, b, and c are natural numbers.

12. Which of the following lists all the functions which have no discontinuities and whose graph has a horizontal asymptote at the x-axis?

A. $f(x)$
B. $k(x)$
C. both $f(x)$ and $h(x)$
D. both $h(x)$ and $k(x)$

$g(x) = \frac{(x+a)(x+c)}{(x+a)(x+b)} = \frac{x+c}{x+b},\ x \neq -a$

$f(x)$ & $k(x)$ have no discontinuities

$f(x)$ has HA $y=0$

$k(x)$ has HA $y=1$

13. Function $g(x)$ has

A. no points of discontinuity
B. two points of discontinuity
C. one vertical asymptote
D. two vertical asymptotes

$g(x) = \frac{(x+a)(x+c)}{(x+a)(x+b)} = \frac{x+c}{x+b}, x \neq -a$

Point of dis at $x = -a$

VA $x = -b$.

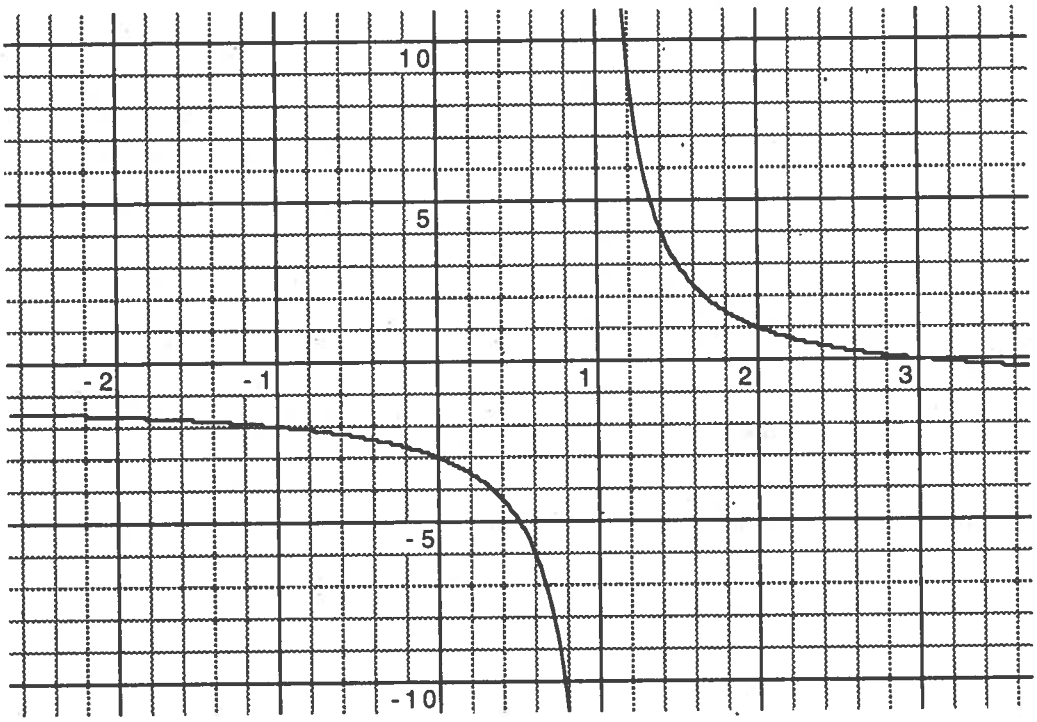
14. The function(s) whose graph(s) has an x-intercept at $x = -a$ is

A. $f(x)$ only
B. $g(x)$ only
C. $f(x)$ and $g(x)$ only
D. some other function or combination of functions

$y = 0$, numerator of function $= 0$

Use the following information to answer the next two questions.

The partial graph of a rational function $y = f(x)$ is shown.



15. The solution of the equation $f(x) - 2.2 = 0$ is

A. $x = -1.5$ **B.** $x = -0.9$
C. $x = 0.8$ **D.** $x = 1.6$

$f(x) = 2.2$

$x = 1.6$

16. The root of the equation $f(2x) = -6$ is

A. 0.3 **B.** 0.6
C. 1.2 **D.** -3.0

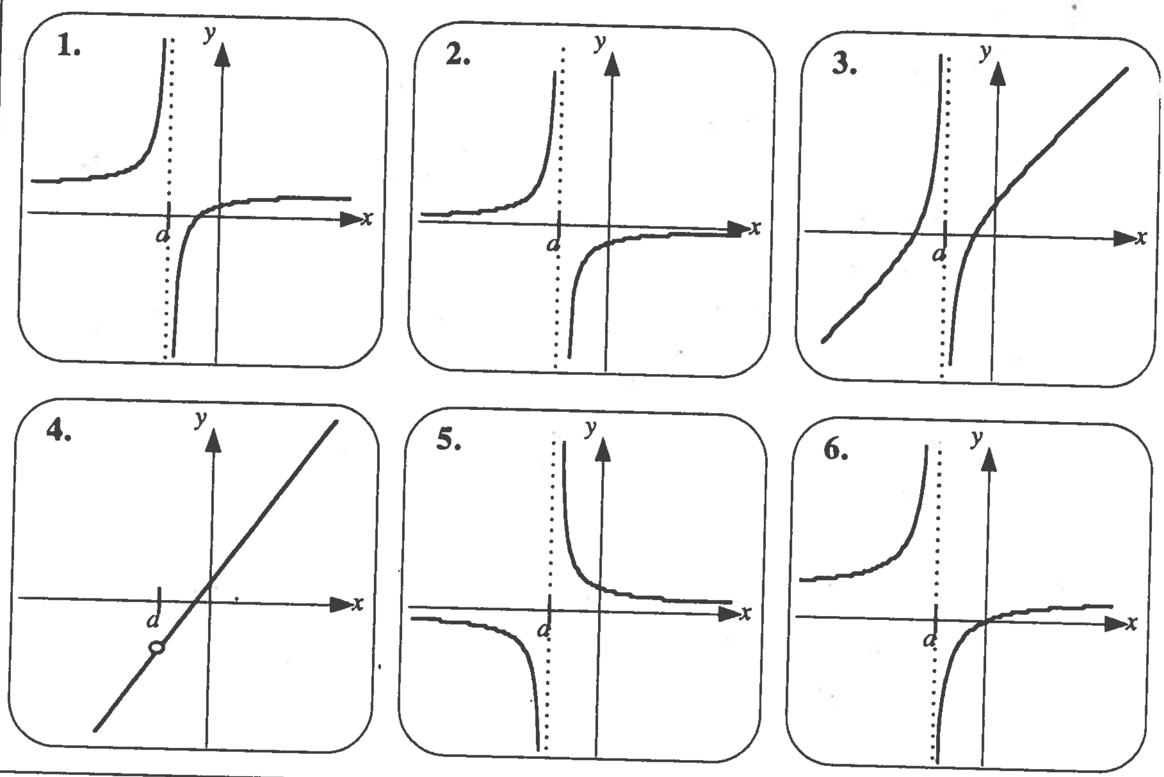
$f(x) = -6 \quad x = 0.6$

$f(2x) = -6 \Rightarrow 2x = 0.6$

$x = 0.3$

Use the following information to answer the next question.

The partial graphs of six rational functions are shown below.



Numerical Response

6. Write the diagram number for the function $y = \dfrac{-2}{x-a}$ in the first box.

Write the diagram number for the function $y = \dfrac{x+2}{x-a}$ in the second box.

Write the diagram number for the function $y = \dfrac{x}{x-a}$ in the third box.

Write the diagram number for the function $y = \dfrac{x^2+2x-ax-2a}{x-a}$ in the last box.

(Record your answer in the numerical response box from left to right.)

2	1	6	4

	VA	HA	y-int	
$y = \frac{-2}{x-a}$	$x = a$	$y = 0$	$\frac{2}{a} < 0$	→ diagram 2.
$y = \frac{x+2}{x-a}$	$x - a$	$y = 1$	$-\frac{2}{a} > 0$	diagram 1
$y = \frac{x}{x-a}$	$x = a$	$y = 1$	0	diagram 6

$y = \dfrac{(x+2)(x-a)}{x-a} = x+2,\ x \neq a$ → line with a point of discontinuity → diagram 4

17. The domain and range of $g(x) = a\sqrt{b(x-h)} + k$, $a > 0, b < 0$, are, respectively

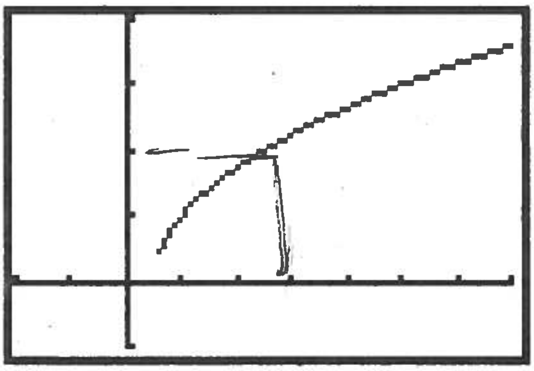
A. $\{x \mid x \geq h, x \in R\}$, $\{g(x) \mid g(x) \leq k, g(x) \in R\}$
B. $\{x \mid x \geq h, x \in R\}$, $\{g(x) \mid g(x) \geq k, g(x) \in R\}$
C. $\{x \mid x \leq h, x \in R\}$, $\{g(x) \mid g(x) \geq -k, g(x) \in R\}$
D. $\{x \mid x \leq h, x \in R\}$, $\{g(x) \mid g(x) \geq k, g(x) \in R\}$

since $b < 0$ $x-h$ must be ≤ 0
$x \leq h$
min. value of $g(x) = a(0) + k$
$= k$
so $g(x) \geq k$.

Use the following information to answer the next three questions.

$f(x)$ is a linear function. The graphing calculator screenshot of the graph of $y = \sqrt{f(x)}$ is shown with window $x:[-2, 7, 1]$, $y[-1, 4, 1]$.

The points $(0.5, 0)$ and $(5, 3)$ lie on the graph of $y = \sqrt{f(x)}$.



18. $f(5)$ is

A. 25 **B.** 9 **C.** $\sqrt{3}$
D. unable to be determined from the given information

$(5, 3)$ lies on $y = \sqrt{f(x)}$
so $(5, 9)$ lies on $y = f(x)$.

19. $f(-5)$ is

A. -11
B. -9
C. -3
D. unable to be determined from the given information

$f(x)$ is ~~linear~~ linear, passing through $(0.5, 0)$ + $(5, 9)$
slope $= \frac{9-0}{5-0.5}$
equation $y = mx + b$
$9 = 2(5) + b$
$b = -1$ $f(x) = 2x - 1$
$f(-5) = 2(-5)$
$= -11$

20. The solution of the equation $\sqrt{f(x-1)} - 2 = 0$ is closest to

A. 1.5 **B.** 2.5
C. 3.5 **D.** 4.5

$\sqrt{f(x)} = 2 \Rightarrow x = 2.5$
$\sqrt{f(x-1)} = 2 \Rightarrow x - 1 = 2.5$ $x = 3.5$

Written Response

Consider the function defined by $f(x) = \dfrac{2x^3 + x^2 - 25x + 12}{(x-3)}$.

- Algebraically determine the coordinates of the point of discontinuity of the graph of the function.

factor $2x^3+x^2-25x+12$ $= (x-3)(2x-1)(x+4)$

$$\begin{array}{r|rrrr} 3 & 2 & 1 & -25 & 12 \\ & & 6 & 21 & -12 \\ \hline & 2 & 7 & -4 & 0 \end{array}$$

$= (x-3)(2x^2+7x-4)$

$= (x-3)(x(2x-1)+4(2x-1))$

$f(x) = \dfrac{(x-3)(2x-1)(x+4)}{(x-3)}$

$= (2x-1)(x+4), \ x \neq 3$

$y = (2(3)-1)(3+4) = 5(7) = 35$

pt@ $(3,35)$

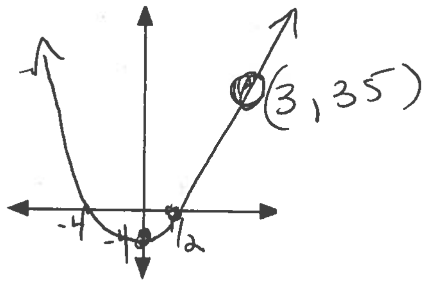
- Describe the relationship between the roots of the equation $\dfrac{2x^3 + x^2 - 25x + 12}{(x-3)} = 0$ and the x-intercepts of the graph of f.

The roots of the equation are the same as the x-int of the grap

- Solve the equation $\dfrac{2x^3 + x^2 - 25x + 12}{(x-3)} = 0$

$\dfrac{(x-3)(2x-1)(x+4)}{x-3} = 0$ $\quad x = -4, \frac{1}{2}$.

- Sketch the graph of the function on the grid showing the intercepts and the point of discontinuity.



y-int
$x=0$
$\dfrac{(0-3)(2(0)-1)(0+4)}{(0-3)} = -4$

Answer Key

Multiple Choice

1. D	2. C	3. C	4. A	5. B	6. D	7. A	8. C
9. D	10. D	11. B	12. A	13. C	14. A	15. D	16. A
17. D	18. B	19. A	20. C				

Numerical Response

1. | 7 | . | 2 | |

2. | 8 | . | 0 | 5 |

3. | 0 | . | 6 | |

4. | 1 | . | 2 | 1 |

5. | 3 | 1 | 2 | 3 |

6. | 2 | 1 | 6 | 4 |

Written Response

- (3, 35)
- The roots of the equation are the same as the x-intercepts of the graph.
- $x = -4, \frac{1}{2}$