

# Assignment

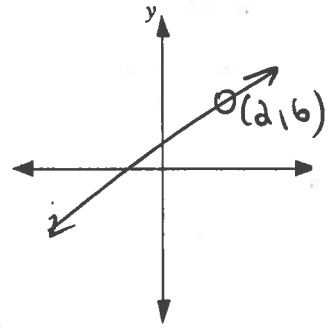
1. Consider the function  $f(x) = \frac{x^2 + 2x - 8}{x - 2}$ .

a) Algebraically determine the point of discontinuity and sketch the graph of the function  $f(x)$ .

$$f(x) = \frac{(x+4)(x-2)}{(x-2)} \quad y = x+4$$

$$= x+4, x \neq 2 \quad y = 2+4$$

$$y = 6 \quad \text{Hole at } (2, 6)$$



b) State the domain and range of  $f$ .

$$x | x \neq 2, x \in \mathbb{R} \quad y | y \neq 6, y \in \mathbb{R}.$$

2. Consider the function  $g(x) = \frac{15 - 2x - x^2}{x + 5}$ .

a) Algebraically determine the point of discontinuity and sketch the graph of the function  $g(x)$ .

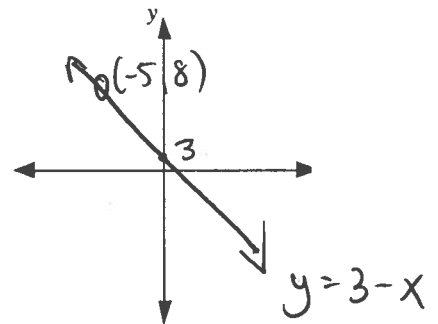
$$g(x) = \frac{(5+x)(3-x)}{x+5} \quad y = 3 - (-5)$$

$$= 3 - x, x \neq -5 \quad = 8$$

$$\text{Hole at } (-5, 8)$$

$$\text{or}$$

$$= -x + 3$$



b) State the domain and range of  $g$ .

$$x | x \neq -5, x \in \mathbb{R} \quad y | y \neq 8, y \in \mathbb{R}.$$

3. Consider the function  $f(x) = \frac{3x^2 - 10x + 3}{3x - 1}$ .

a) Sketch the graph of the function  $f(x)$  and determine the point of discontinuity.

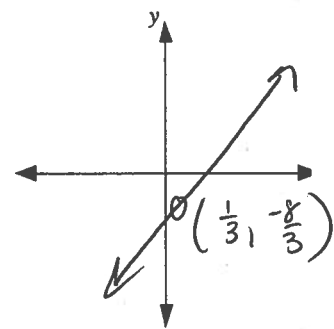
$$3x^2 - 10x + 3 \quad f(x) = \frac{(x-3)(3x-1)}{3x-1}$$

$$= 3x^2 - 9x - x + 3$$

$$= 3x(x-3) - 1(x-3)$$

$$= (x-3)(3x-1)$$

$$= x-3, x \neq 1/3.$$



$$y = \frac{1}{3} - 3 = -\frac{8}{3} \quad \text{Hole @ } \left(\frac{1}{3}, -\frac{8}{3}\right)$$

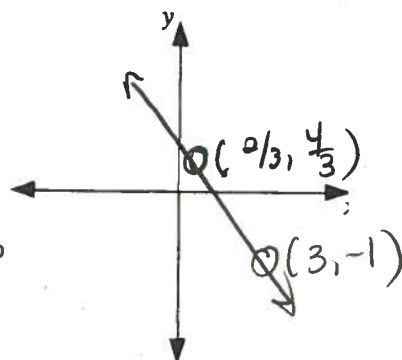
b) State the domain and range of  $f$ .

$$x | x \neq \frac{1}{3}, x \in \mathbb{R} \quad y | y \neq -\frac{8}{3}, y \in \mathbb{R}$$

4. Algebraically determine the point(s) of discontinuity

of the graph of  $g(x) = \frac{-3x^3 + 17x^2 - 28x + 12}{3x^2 - 11x + 6}$ .

Determine the domain and range of the function and sketch the graph on the grid.



factor  $-3x^2 + 17x^2 - 28x + 12$   
 $= -(3x^2 - 17x^2 + 28x - 12)$

factor  $3x^2 - 11x + 6$   
 $= 3x^2 - 9x - 2x + 6$   
 $= 3x(x-3) - 2(x-3)$   
 $= (x-3)(3x-2)$

$$\begin{array}{r|rrrr} 2 & 3 & -17 & 28 & -12 \\ & & 6 & -22 & 12 \\ \hline & 3 & -11 & 6 & 0 \end{array}$$

$= -(x-2)(3x^2 - 11x + 6)$   
 $= -(x-2)(x-3)(3x-2)$

$g(x) = \frac{-(x-2)(x-3)(3x-2)}{(x-3)(3x-2)} = -(x-2), x \neq \frac{2}{3}, 3$

$x | x \neq \frac{2}{3}, 3, x \in \mathbb{R}$   
 $y | y \neq \frac{4}{3}, -1, y \in \mathbb{R}$

$x = \frac{2}{3} \quad y = -(x-2)$   
 $y = -(\frac{2}{3} - 2)$   
 $y = \frac{4}{3}$

$x = 3 \quad y = -(x-2)$   
 $y = -(3-2)$   
 $y = -1$

5. Algebraically determine the point(s) of discontinuity and the domain and range for the

function  $f(x) = \frac{2x^3 - 5x^2 - 9x + 18}{2x - 3}$ . Sketch the graph on the grid.

factor  $2x^3 - 5x^2 - 9x + 18$

$$\begin{array}{r|rrrr} -2 & 2 & -5 & -9 & 18 \\ & & -14 & 18 & -18 \\ \hline & 2 & -9 & 9 & 0 \end{array}$$

$= (x+2)(2x^2 - 9x + 9)$

$= (x+2)[2x^2 - 6x - 3x + 9]$

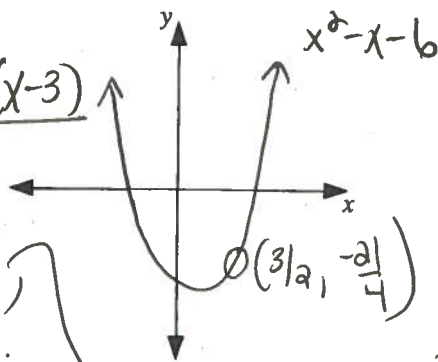
$= (x+2)[2x(x-3) - 3(x-3)]$

$= (x+2)(2x-3)(x-3)$

$f(x) = \frac{(x+2)(2x-3)(x-3)}{2x-3}$

$= (x+2)(x-3)$   
 $x \neq \frac{3}{2}$

$y = (\frac{3}{2} + 2)(\frac{3}{2} - 3)$   
 $= \frac{-21}{4}$



$f(x) = x^2 - x - 6$   
 $x \neq \frac{3}{2}$

Hole:  $(\frac{3}{2}, \frac{-21}{4})$   
 $x | x \neq \frac{3}{2}, x \in \mathbb{R}$   
 $y | y \neq \frac{-21}{4}, y \in \mathbb{R}$

Multiple Choice

6. The range of the function  $f(x) = \frac{2x^2 + 7x + 3}{x + 3}$  is

- A.  $\{y \mid y \neq -3, y \in \mathbb{R}\}$
- B.  $\{y \mid y \neq -4, y \in \mathbb{R}\}$
- C.  $\{y \mid y \neq -5, y \in \mathbb{R}\}$
- D.  $\{y \mid y \in \mathbb{R}\}$

$$f(x) = \frac{(x+3)(2x+1)}{x+3}$$

$$= 2x+1, x \neq -3$$

$$\hookrightarrow 2(-3)+1 = -5$$

$$y \mid y \neq -5, y \in \mathbb{R}$$

7. The equation of a rational function can be written in the form  $y = x - p, x \neq -3$ . The graph of the rational function has a point of discontinuity at  $(-3, 10)$ . The value of  $p$  is

- A. 13
- B. 7
- C. -7
- D. -13

$$y = x - p$$

$$10 = -3 - p$$

$$p = -3 - 10 = -13$$

Use the following information to answer the next three questions.

Leo was given a function in the form  $f(x) = \frac{ax^3 + bx^2 + cx + d}{2x + 5}$ . He was told that the numerator could be expressed in the factored form  $(2x + 5)(x^2 - p)$ , and that the graph of the function had a point of discontinuity at  $(q, \frac{9}{4})$ .

8. The value of  $q$  is

- A.  $\frac{2}{5}$
- B.  $\frac{5}{2}$
- C.  $-\frac{2}{5}$
- D.  $-\frac{5}{2}$

9. The value of  $p$  is

- A. 4
- B. -4
- C.  $\frac{17}{2}$
- D.  $-\frac{17}{2}$

nonpermissible value =  $-\frac{5}{2}$  so  $q = -\frac{5}{2}$

$$f(x) = \frac{(2x+5)(x^2-p)}{2x+5} = x^2 - p, x \neq -\frac{5}{2}$$

replace  $-\frac{5}{2}$  into  $x^2 - p$  to get

$$\frac{9}{4} = \left(-\frac{5}{2}\right)^2 - p \quad p = \frac{25}{4} - \frac{9}{4} = 4$$

10. The value of  $a + b + c + d$  is

- A. 7
- B. -21
- C. 23
- D. none of the above

numerator  $(2x+5)(x^2-4)$

$$= 2x^3 - 8x + 5x^2 - 20$$

$$= 2x^3 + 5x^2 - 8x - 20$$

$a=2 \quad b=5 \quad c=-8 \quad d=-20$

$$a+b+c+d = -21$$

11. The graph of  $y = \frac{x^2 + bx + c}{x - d}$  is a straight line with a point of discontinuity  $(2, 5)$ .

The value of  $c$  is

- A. -6
- B. -3
- C. 3
- D. 6

*\* not clear*  
 nonpermissible value =  $d$  so  $d = 2$   
 numerator has a factor  $x - 2$   
 constant term in numerator is  $= c$ ,  
 so product of constant terms in  
 factors of numerator  $= c$

$$y = \frac{(x-2)(x-\frac{1}{2}c)}{x-2}$$

$$y = x - \frac{1}{2}c$$

$$5 = 2 - \frac{1}{2}c$$

$$\frac{1}{2}c = -3$$

$$c = -6$$

**Numerical Response**

12. The function  $f(x) = \frac{x^2 - 4x - k}{x + 3}$  has a point of discontinuity. The value of  $k$  is

(Record your answer in the numerical response box from left to right.)

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$$f(x) = \frac{(x+3)(x - \frac{1}{3}k)}{(x+3)} = \frac{x^2 + 3x - \frac{1}{3}kx - k}{x+3}$$

$$= 3 - \frac{1}{3}k = -4$$

$$7 = \frac{1}{3}k \quad k = 21$$

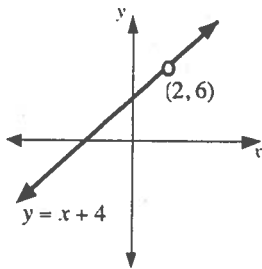
**Answer Key**

See graphs below for questions 1a), 2a), 3a), 4, and 5.

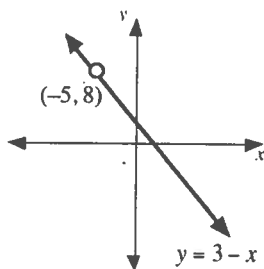
- 1. a) (2, 6)      b) Domain:  $\{x | x \neq 2, x \in R\}$       Range:  $\{y | y \neq 6, y \in R\}$
- 2. a) (-5, 8)      b) Domain:  $\{x | x \neq -5, x \in R\}$       Range:  $\{y | y \neq 8, y \in R\}$
- 3. a)  $(\frac{1}{3}, -\frac{8}{3})$       b) Domain:  $\{x | x \neq \frac{1}{3}, x \in R\}$       Range:  $\{y | y \neq -\frac{8}{3}, y \in R\}$
- 4. Points of Discontinuity  $(\frac{2}{3}, \frac{4}{3}), (3, -1)$ . Domain:  $\{x | x \neq \frac{2}{3}, 3, x \in R\}$       Range:  $\{y | y \neq -1, \frac{4}{3} y \in R\}$
- 5. Point of Discontinuity  $(\frac{3}{2}, -\frac{21}{4})$ . Domain:  $\{x | x \neq \frac{3}{2}, x \in R\}$       Range:  $\{y | y \geq -\frac{25}{4}, y \neq -\frac{21}{4} y \in R\}$
- 6. C      7. D      8. D      9. A      10. B      11. A      12. 

2	1		
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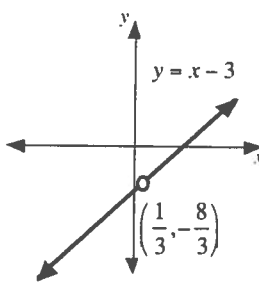
1a)  $f(x) = \frac{x^2 + 2x - 8}{x - 2}$



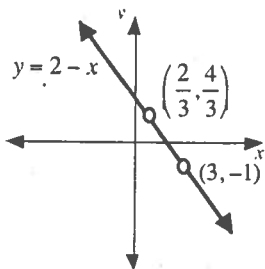
2a)  $g(x) = \frac{15 - 2x - x^2}{x - 2}$



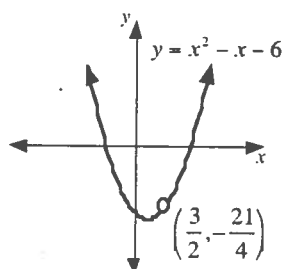
3a)  $f(x) = \frac{3x^2 - 10x + 3}{3x - 1}$



4.  $g(x) = \frac{-3x^3 + 17x^2 - 28x + 12}{3x^2 - 11x + 6}$



5.  $f(x) = \frac{2x^3 - 5x^2 - 9x + 18}{2x - 3}$



$$y = \frac{(x-2)(x-n)}{x-2}$$

$$5 = \frac{(2-2)(2-n)}{(2-2)}$$

$$5 = 2 - n$$

$$-3 = n$$

$$x = -3$$