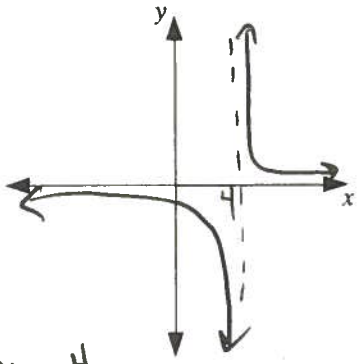
Assignment

1. In the following examples use a graphing calculator to sketch the graph of the function and determine the

i) equation(s) of the vertical asymptote(s) **ii)** equation of the horizontal asymptote

iii) domain and range of the function **iv)** x and y-intercepts of the graph

a) $f(x) = \dfrac{5}{x-4}$



i) $x = 4$

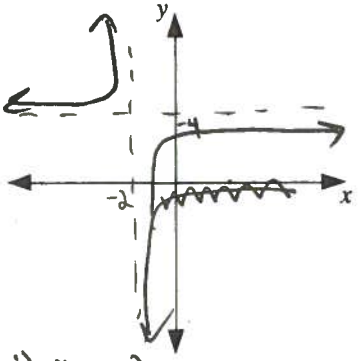
ii) $y = 0$

iii) $x \mid x \neq 4, x \in \mathbb{R}$

$y \mid y \neq 0, y \in \mathbb{R}$

iv) no x-int, y-int $= -\frac{5}{4}$

b) $f(x) = \dfrac{4x+5}{x+2}$



i) $x = -2$

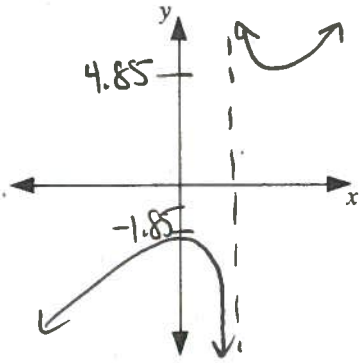
ii) $y = 4$

iii) $x \mid x \neq 2, x \in \mathbb{R}$

$y \mid y \neq 4, y \in \mathbb{R}$

iv) x-int $= -\frac{5}{4}$, y-int $= \frac{5}{2}$.

c) $f(x) = \dfrac{x^2+9}{2x-3}$



i) $x = \frac{3}{2}$

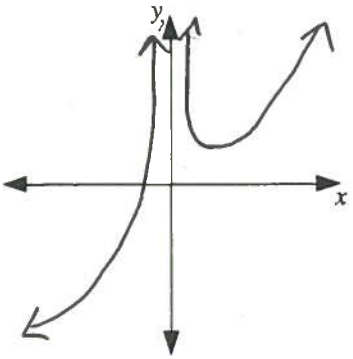
ii) none

iii) $x \mid x \neq \frac{3}{2}, x \in \mathbb{R}$

$y \mid y \leq -1.85$ or $y \geq 4.85 \; y \in \mathbb{R}$

iv) no x-int, y-int $= -3$

d) $f(x) = \dfrac{x^3+1}{x^2}$



i) $x = 0$

ii) none

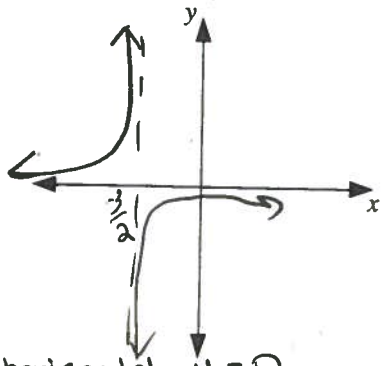
iii) $x \mid x \neq 0, x \in \mathbb{R}$

$y \in \mathbb{R}$

iv) x-int $= -1$, no y-int

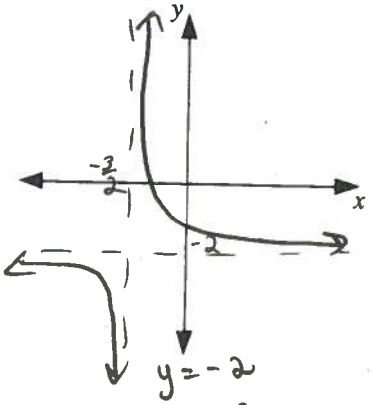
2. In each of the following examples use a graphing calculator to sketch the graph of the function and state the equations of any horizontal and vertical asymptotes on the graph.

a) $y = \dfrac{-4}{2x+3}$



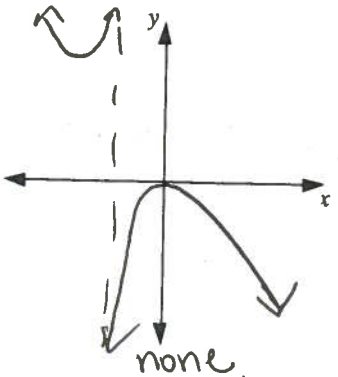
horizontal $y=0$
vertical $x=-\frac{3}{2}$

b) $y = \dfrac{-4x}{2x+3}$



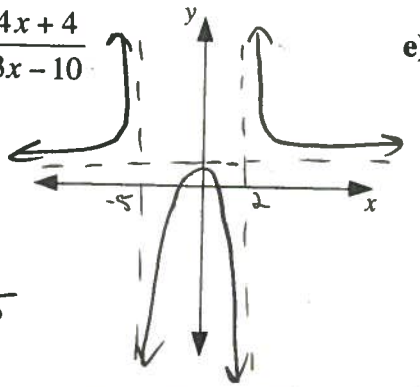
$y=-2$
$x=-\frac{3}{2}$

c) $y = \dfrac{-4x^2}{2x+3}$



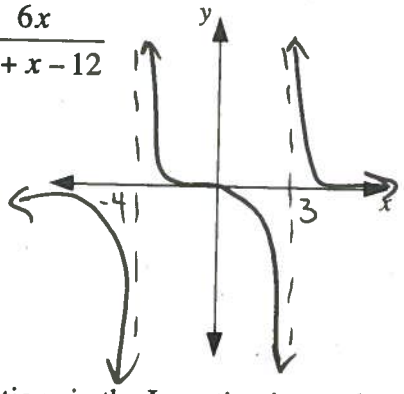
none
$x=-3/2$.

d) $y = \dfrac{x^2+4x+4}{x^2+3x-10}$



hori: $y=1$
vert $x=-5$
$x=2$

e) $y = \dfrac{6x}{x^2+x-12}$



hori: $y=0$
$x=-4$
$x=3$

3. Consider the twelve graphs and equations of rational functions in the Investigation and in Assignment Questions #1 and #2.

a) The line $y = 0$ is a horizontal asymptote in four of the graphs. Determine a relationship between the degree of the numerator and the degree of the denominator that exists in these four examples and not in the other eight examples.

The degree of the numerator is less than the degree of the denominator.

b) There is no horizontal asymptote in four of the graphs. Determine a relationship between the degree of the numerator and the degree of the denominator that exists in these four examples and not in the other eight examples.

The degree of the numerator is greater than the degree of the denominator.

c) A line other than $y = 0$ is a horizontal asymptote in four of the graphs.

i) Determine a relationship between the degree of the numerator and the degree of the denominator that exists in these four examples and not in the other eight examples.

The degree of the numerator is equal to the degree of the denominator

ii) Suggest a rule that will determine the value of k if the horizontal asymptote has equation $y = k$.

k = the leading coefficient of the numerator ÷ by the leading coefficient of the denominator.

4. Algebraically determine the equations of the asymptotes of the graph of each of the following functions.

a) $f(x) = \frac{2}{x+3}$

vertical asymtote

$x+3=0$, $x=-3$

since degree of numerator is less than degree of denominator

horizontal asymptote → $y=0$

b) $f(x) = \frac{x^2+5x+6}{x+6}$

vertical asymtote $x=-6$

since degree of numerator is greater then degree of denominator there is no horizontal asymptote.

c) $f(x) = \frac{4x}{1-4x}$

$x = \frac{1}{4}$

degree of numerator = degree of denominator

horizontal asymptote $y=-1$

$\left(\frac{4}{-4}\right) = -1$

d) $f(x) = \frac{x}{x^2-4}$

$x^2-4 \rightarrow (x-2)(x+2)$ $x = \pm 2$

$x=-2$, $x=2$

since degree of numerator is less than degree of denominator

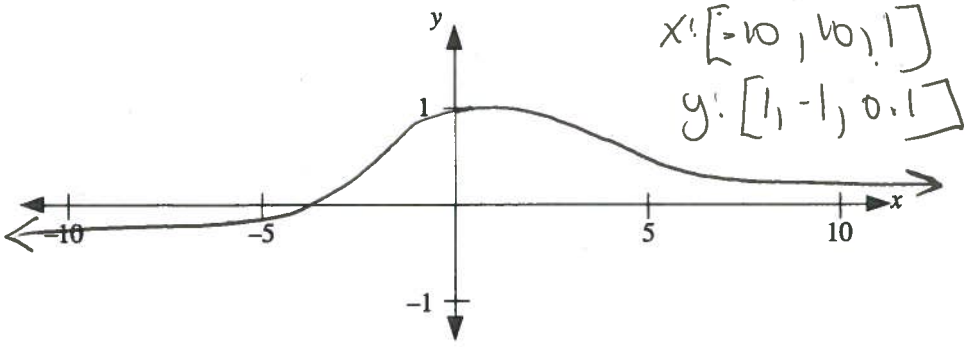
horizontal asymptote $y=0$

5. Students were asked to sketch a graph of a function with one x-intercept and a horizontal asymptote at the x-axis.

Tricia claimed that this could not be done as a graph cannot cross an asymptote.

Mandy claimed it is possible for a graph to cross the x-axis, but still get closer and closer to the x-axis as $x \rightarrow \pm\infty$.

Investigate the students' claims by sketching the graph of $y = \frac{x+4}{x^2+4}$ on the grid provide



x: [-10, 10, 1]

y: [1, -1, 0.1]

Multiple Choice

6. The horizontal asymptote of the graph of the function $g(x) = \frac{-7x+2}{3x+2}$ is

A. $y = 0$ **B.** $y = -\frac{7}{3}$ **C.** $y = -\frac{3}{7}$ **D.** $x = -\frac{2}{3}$

7. Consider the rational function $f(x) = \frac{ax+5}{7x-b}$, where a and b are natural numbers. The vertical asymptote and the horizontal asymptote of the graph of $f(x)$, respectively, are

A. $x = b, y = a$ **B.** $x = \frac{7}{b}, y = -\frac{5}{a}$ **C.** $x = \frac{b}{7}, y = \frac{a}{7}$ **D.** $x = \frac{b}{7}, y = -\frac{a}{7}$

$7x - b = 0 \quad x = \frac{b}{7}$

Numerical Response

8. The graph of the function $f(x) = \frac{3x-8}{x-8}$ has a vertical asymptote with equation $x = p$ and a horizontal asymptote with equation $y = q$. The value of pq is _____.

(Record your answer in the numerical response box from left to right.)

2	4		

$y = 3 \quad p = 8 \quad pq = 8(3) = 24$

$x = 8 \quad q = 3$

9. The graph of the function $g(x) = \frac{5x}{10x^2 - 27x + 5}$ has asymptotes with equations $x = a$, $x = b$, and $y = c$. To the nearest tenth, the value of $a + b + c$ is _____.

(Record your answer in the numerical response box from left to right.)

2	.	7	

$10x^2 - 27x + 5$

$= 10x^2 - 25x - 2x + 5$

$= 5x(2x-5) - 1(2x-5)$

$(2x-5)(5x-1)$

$g(x) = \frac{5x}{(2x-5)(5x-1)}$

vert. asy $x = \frac{5}{2}, x = \frac{1}{5}$

$y = 0$

$a = \frac{5}{2} \quad b = \frac{1}{5} \quad c = 0$

$a + b + c = \frac{5}{2} + \frac{1}{5} + 0$

$= 2.7$

Answer Key

1. a)

i) $x = 4$

ii) $y = 0$

iii) $\{x \mid x \neq 4, x \in R\}$ $\{y \mid y \neq 0, y \in R\}$

iv) x-intercept = none; y-intercept = $-\frac{5}{4}$

b)

$x = -2$

$y = 4$

$\{x \mid x \neq -2, x \in R\}$ $\{y \mid y \neq 4, y \in R\}$

x-intercept = $-\frac{5}{4}$; y-intercept = $\frac{5}{2}$

c)

$x = \frac{3}{2}$

none

$\{x \mid x \neq \frac{3}{2}, x \in R\}$ $\{y \mid y \leq -1.85 \text{ or } y \geq 4.85, y \in R\}$

x-intercept = none; y-intercept = -3

d)

$x = 0$

none

$\{x \mid x \neq 0, x \in R\}$ $\{y \mid y \in R\}$

x-intercept = -1; y-intercept = none

2. a) $y = 0, x = -\frac{3}{2}$ **b)** $y = -2, x = -\frac{3}{2}$ **c)** none, $x = -\frac{3}{2}$

d) $y = 1, x = -5, x = 2$ **e)** $y = 0, x = -4, x = 3$

3. a) The degree of the numerator is less than the degree of the denominator.

b) The degree of the numerator is greater than the degree of the denominator.

c) i) The degree of the numerator is equal to the degree of the denominator.

ii) k = the leading coefficient of the numerator divided by the leading coefficient of the denominator.

4. a) $y = 0, x = -3$ **b)** $x = -6$ **c)** $y = -1, x = \frac{1}{4}$ **d)** $y = 0, x = -2, x = 2$

5. Mandy is correct **6.** B **7.** D **8.** 24 **9.** 2.7