Name key.

Math 20-1 Radical Expressions Operations

1. Order the radical expressions in each set from least to greatest.

a) $\sqrt{2}, -\sqrt{9}, \sqrt{5}, \sqrt{8}$

b) $\sqrt{2}, \sqrt{16}, \sqrt[3]{27}, \sqrt[5]{32}$

2. Use your knowledge of radicals to show that $\sqrt{800}$ is equivalent to

a) $10\sqrt{8}$ $\sqrt{800} = \sqrt{100 \cdot 8} = \sqrt{100} \cdot \sqrt{8} = 10\sqrt{8}$

b) $20\sqrt{2}$ $= \sqrt{400 \cdot 2} = 20\sqrt{2}$

c) $5\sqrt{32}$ $= \sqrt{25 \cdot 32} = 5\sqrt{32}$

Which one of these equivalent expressions would be $\sqrt{800}$ expressed in lowest terms? Explain.

b) → the radicant is in simplest form.

3. Express each radical in simplest form as a mixed radical, $a\sqrt{b}$.

a) $\sqrt{40} = \sqrt{4 \cdot 10}$
$= 2\sqrt{10}$

b) $2\sqrt{135} = 2\sqrt{9 \cdot 15}$
$= 2 \cdot 3\sqrt{15}$
$= 6\sqrt{15}$

c) $-\sqrt[3]{250}$
$-\sqrt[3]{125 \cdot 2}$
$-5\sqrt[3]{2}$

d) $\sqrt{1000a^5b^6}, a \geq 0, b \geq 0$
$\sqrt{100 \cdot 10 \cdot a^4 \cdot a \cdot b^6}$
$= 10a^2b^3\sqrt{10a}$

4. Express each as an entire radical.

a) $-2\sqrt{20}$
$-\sqrt{2^2 \cdot 20} = -\sqrt{80}$

b) $3mn^3\sqrt{5m} = \sqrt{3^2m^2(n^3)^2 \cdot 5m}$
$= \sqrt{45m^3n^6}$

c) $2\sqrt[5]{3}$
$\sqrt[5]{2^5 \cdot 3} = \sqrt[5]{96}$

d) $5\sqrt{x+1}$
$\sqrt{5^2(x+1)} = \sqrt{25x+25}$

5. Simplify $\sqrt{25} - \sqrt{36} \times \sqrt[3]{8} + \sqrt[4]{16}$.
$5 - 6 \cdot 2 + 2$
$5 - 12 + 2 = -5$

6. True or False. Correct all false statements.

a) $-m^2 = m^2$
False
$-1(m)^2$

b) $\sqrt{(-9)^2} = -9$
↓
$\sqrt{81} = -9$
$9 \neq -9$
False.

c) $\sqrt{r^2} = \pm r$
true.

7. The horizontal distance, d, in kilometers that can be viewed from a given height, h, in metres above the ground is given by $d = 80\sqrt{2h}$. What distance can be seen from a height of 100 meters? Express the answer in simplest terms.

$d = 80\sqrt{2(100)}$

$d = 80\sqrt{200}$ (100 · 2)

$\rightarrow d = 80 \cdot 10\sqrt{2}$

$= 800\sqrt{2}$

8. Simplify each and express in the form $a\sqrt{b}$.

a) $3\sqrt{2} - 2\sqrt{3} + 4\sqrt{2} + 5\sqrt{3}$

$7\sqrt{2} + 3\sqrt{3}$

b) $6\sqrt{45} - 4\sqrt{125}$ (9·5, 25·5)

$= 6 \cdot 3\sqrt{5} - 4 \cdot 5\sqrt{5}$

$= 18\sqrt{5} - 20\sqrt{5}$

c) $7\sqrt{12} - 8\sqrt{75} - 3\sqrt{27}$ (4·3, 25·3, 9·3)

$= 7 \cdot 2\sqrt{3} - 8 \cdot 5\sqrt{3} - 3 \cdot 3\sqrt{3}$

$= 14\sqrt{3} - 40\sqrt{3} - 9\sqrt{3}$

$= -35\sqrt{3}$

d) $\sqrt{8} - \sqrt[3]{8} + \sqrt[3]{54}$ (4·2, 2, 27·2)

$2\sqrt{2} - 2 + 3\sqrt[3]{2}$

e) $5\sqrt{2} \cdot 2\sqrt{5}$

$10\sqrt{10}$

f) $5\sqrt{3}(2\sqrt{6} - 4\sqrt{3})$

$10\sqrt{18} - 20\sqrt{9}$ (9·2)

$10 \cdot 3\sqrt{2} - 20(3)$

$30\sqrt{2} - 60$

g) $(4\sqrt{7} + 3\sqrt{5})(2\sqrt{7} - \sqrt{5})$

$8(7) - 4\sqrt{35} + 6\sqrt{35} - 3(5)$

$56 + 2\sqrt{35} - 15$

$41 + 2\sqrt{35}$

h) $(\sqrt{7a} - \sqrt{11a})(\sqrt{7a} + \sqrt{11a})$

$7a + \sqrt{77a^2} - \sqrt{77a^2} - 11a$

$= -4a$

i) $(2\sqrt{5} - 5)^2$

$(2\sqrt{5} - 5)(2\sqrt{5} - 5)$

$4(5) - 10\sqrt{5} - 10\sqrt{5} + 25$

$20 - 20\sqrt{5} + 25$

$45 - 20\sqrt{5}$

k) $\frac{\sqrt{12}}{\sqrt{6}} = \sqrt{\frac{12}{6}} = \sqrt{2}$

9. Determine the exact value in simplest form for the area of each figure.

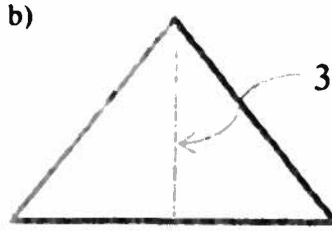
a)

$(5\sqrt{3})(7\sqrt{3}+2\sqrt{5})$

$= 35(3) + 10\sqrt{15}$

$= 105 + 10\sqrt{15}$

b)



$= \dfrac{(3\sqrt{5})(10\sqrt{5}-\sqrt{8})}{2}$

$= \dfrac{30(5) - 3\sqrt{40}}{2}$ (4·10) $= \dfrac{150 - 6\sqrt{10}}{2}$

$= 75 - 3\sqrt{10}$

10. Express each in simplest form with a rational denominator.

a) $\dfrac{3}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{3\sqrt{5}}{5}$

b) $\dfrac{3\sqrt{35}}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{3\sqrt{245}}{7} = \dfrac{3\sqrt{49 \cdot 5}}{7}$

$= \dfrac{21\sqrt{5}}{7} = 3\sqrt{5}$

(should have just ÷ $\dfrac{3\sqrt{35}}{\sqrt{7}} = 3\sqrt{\dfrac{35}{7}} \to 3\sqrt{5}$)

c) $\dfrac{10\sqrt{6}}{\sqrt{12}} = 10\sqrt{\dfrac{6}{12}}$

$= 10\sqrt{\dfrac{1}{2}}$

$= \dfrac{10}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$

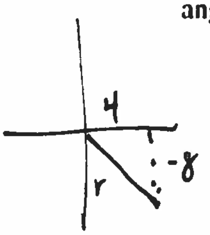
d) $\dfrac{(3\sqrt{2}+7)}{(4\sqrt{3}-2)} \cdot \dfrac{(4\sqrt{3}+2)}{(4\sqrt{3}+2)}$

$= \dfrac{12\sqrt{6} + 6\sqrt{2} + 28\sqrt{3} + 14}{16(3) - 4}$

$= \dfrac{12\sqrt{6} + 6\sqrt{2} + 28\sqrt{3} + 14}{32}$

$= \dfrac{6\sqrt{6} + 3\sqrt{2} + 14\sqrt{3} + 7}{16}$

11. Determine the exact value of the primary trig ratios given the point $(4,-8)$ on the terminal arm of an angle in standard position. Radicals must be in simplest form with a rational denominator.



$x^2 + y^2 = r^2$

$4^2 + (-8)^2 = r^2$

$\sqrt{80} = r$

$r = 4\sqrt{5}$

$\sin\theta = \dfrac{y}{r} = \dfrac{-8}{4\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{-8\sqrt{5}}{20} = \dfrac{-4\sqrt{5}}{5}$

$\cos\theta = \dfrac{x}{r} = \dfrac{4}{4\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-8}{4} = -2$

12. Use the quadratic formula to determine the exact roots of the equation $x^2 - 4x = 8$.

$x^2 - 4x - 8 = 0$

A B C

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{16+32}}{2} = \dfrac{4 \pm \sqrt{48}}{2} = \dfrac{4 \pm \sqrt{16 \cdot 3}}{2}$

$= \dfrac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

Name: Key

Operations on Radicals Quiz (AN2)

1. $12x^2y\sqrt{5x}$ written as an entire radical is

$\sqrt{720x^5y^2}$

2. When written in simplest form, $\sqrt[3]{875}$ can be written as $a\sqrt[3]{b}$. The value of $a + b$ is

$5\sqrt[3]{7} = 5 + 7 = \underline{12}$

3. Simplify $3\sqrt{175} + 6\sqrt{63}$

$15\sqrt{7} + 18\sqrt{7} = 33\sqrt{7}$

4. The product of $10 - 3\sqrt{5}$ and its conjugate, to the nearest whole number, is

$(10 - 3\sqrt{5})(10 + 3\sqrt{5})$

$= 100 - 45 = \underline{55}$

5. Fully simplify $\left(\frac{\sqrt{8}+3}{2}\right)\left(\frac{4-\sqrt{2}}{\sqrt{3}}\right)$

$= \frac{4\sqrt{8} - \sqrt{16} + 12 - 3\sqrt{2}}{2\sqrt{3}}$

$= \frac{8\sqrt{2} - 4 + 12 - 3\sqrt{2}}{2\sqrt{3}}$

$= \frac{5\sqrt{2} + 8}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{6} + 8\sqrt{3}}{6}$

6. Fully simplify $\frac{2\sqrt{5}}{\sqrt{10}+3\sqrt{2}} \cdot \frac{(\sqrt{10} - 3\sqrt{2})}{(\sqrt{10} - 3\sqrt{2})}$

$= \frac{2\sqrt{50} - 6\sqrt{10}}{10 - 18} = \frac{10\sqrt{2} - 6\sqrt{10}}{-8}$

$= \frac{-5\sqrt{2} + 3\sqrt{10}}{4}$ or $\frac{3\sqrt{10} - 5\sqrt{2}}{4}$.