

Assignment

1. Determine a) the third term of $(x+y)^{15}$

$$\begin{aligned}
 k &\rightarrow 2 & t_{k+1} &= n C_k x^{n-k} y^k \\
 n &= 15 & t_3 &= 15 C_2 x^{13} y^2 \\
 & & t_3 &= 105 x^{13} y^2
 \end{aligned}$$

b) the middle term of $(p+q)^{12}$ term 7.

$$\begin{aligned}
 k &= 6 & t_7 &= 12 C_6 p^6 q^6 \\
 n &= 12 & t_7 &= 924 p^6 q^6
 \end{aligned}$$

2. Use the binomial theorem to write the expansion of $(x+y)^8$.

$$\begin{aligned}
 (x+y)^8 &= 8 C_0 x^8 + 8 C_1 x^7 y + 8 C_2 x^6 y^2 + 8 C_3 x^5 y^3 + 8 C_4 x^4 y^4 + 8 C_5 x^3 y^5 + 8 C_6 x^2 y^6 + \\
 &\quad 8 C_7 x y^7 + 8 C_8 y^8 \\
 &= x^8 + 8x^7 y + 28x^6 y^2 + 56x^5 y^3 + 70x^4 y^4 + 56x^3 y^5 + 28x^2 y^6 + 8xy^7 + y^8
 \end{aligned}$$

3. Find the indicated term of each expansion.

a) the fifth term of $(a-b)^5$

$$\begin{aligned}
 x &\rightarrow a & t_{k+1} &= n C_k x^{n-k} y^k \\
 y &\rightarrow -b & t_5 &= 5 C_4 a^1 (-b)^4 \\
 k &\rightarrow 4 & t_5 &= 5ab^4 \\
 n &\rightarrow 5
 \end{aligned}$$

b) the second term of $(x-2)^6$

$$\begin{aligned}
 x &\rightarrow x & t_2 &= 6 C_1 x^5 (-2)^1 \\
 y &\rightarrow -2 & &= 6x^5 (-2) \\
 k &\rightarrow 1 & &= -12x^5 \\
 n &\rightarrow 6
 \end{aligned}$$

c) the third term of $(3x+2y)^9$

$$\begin{aligned}
 x &\rightarrow 3x & t_3 &= 9 C_2 (3x)^7 (2y)^2 \\
 y &\rightarrow 2y & &= 9 C_2 (3^7 x^7) 2^2 y^2 \\
 k &\rightarrow 2 & &= 36(2187)x^7(4)y^2 \\
 n &\rightarrow 9 & &= 314928x^7y^2
 \end{aligned}$$

d) the fourth term of $(a^2-2a)^7$

$$\begin{aligned}
 x &\rightarrow a^2 & t_4 &= 7 C_3 (a^2)^4 (-2a)^3 \\
 y &\rightarrow -2a & &= 35a^8 (-8a^3) \\
 k &\rightarrow 3 & &= \underline{\underline{-280a^{11}}} \\
 n &\rightarrow 7
 \end{aligned}$$

e) the middle term of $(2-\frac{x}{2})^6$

term 4.

$$\begin{aligned}
 x &\rightarrow 2 & t_4 &= 6 C_3 (2)^3 \left(-\frac{1}{2}x\right)^3 \\
 y &\rightarrow -\frac{1}{2}x & &= 20(8)\left(-\frac{1}{2}\right)^3 x^3 \\
 k &\rightarrow 3 & &= \underline{\underline{-20x^3}} \\
 n &\rightarrow 6
 \end{aligned}$$

4. Expand and write in the simplest form:

$$\begin{aligned} \text{a) } (2x+y)^3 &= {}_3C_0(2x)^3 + {}_3C_1(2x)^2y + {}_3C_2(2x)y^2 + {}_3C_3y^3 \\ &= 1(8x^3) + 3(4x^2)y + 3(2x)y^2 + 1y^3 \\ &= 8x^3 + 12x^2y + 6xy^2 + y^3 \end{aligned}$$

$$\begin{aligned} \text{b) } \left(x - \frac{1}{x}\right)^4 &= {}_4C_0x^4 + {}_4C_1x^3\left(-\frac{1}{x}\right)^1 + {}_4C_2x^2\left(-\frac{1}{x}\right)^2 + {}_4C_3x\left(-\frac{1}{x}\right)^3 + {}_4C_4\left(-\frac{1}{x}\right)^4 \\ &= 1x^4 + 4(-x)^2 + 6(x^0) + 4\left(-\frac{1}{x^2}\right) + 1\left(\frac{1}{x^4}\right) \\ &= x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4} \end{aligned}$$

5. Find the indicated term of each expansion.

a) the term in x^3 in $(1-2x)^{12}$

$$\begin{aligned} x &\rightarrow 1 & t_{k+1} &= {}_n C_k x^{n-k} y^k \\ y &\rightarrow -2x & &= {}_{12} C_k (1)^{12-k} (-2x)^k \\ k &\rightarrow k & &= {}_{12} C_k (1)(-2)^k x^k \\ n &\rightarrow 12 & & \end{aligned}$$

term in x^3 $k=3$

$$\begin{aligned} t_4 &= {}_{12} C_3 (1)(-2)^3 x^3 \\ &= -1760x^3 \end{aligned}$$

b) the term in x^4y^3 in $(3x-y)^7$

$$\begin{aligned} x &\rightarrow 3x & t_{k+1} &= {}_7 C_k (3x)^{7-k} (-y)^k \\ y &\rightarrow -y & &= {}_7 C_k 3^{7-k} x^{7-k} (-1)^k y^k \\ k &\rightarrow k & & \\ n &\rightarrow 7 & & \end{aligned}$$

term in x^4y^3 $k=3$

$$\begin{aligned} t_4 &= {}_7 C_4 (3^{7-3})(-1)^3 x^4 y^3 \\ &= {}_7 C_4 (3^4)(-1)^3 x^4 y^3 \\ &= -2835x^4y^3 \end{aligned}$$

6. a) One term in the expansion of

$(x+a)^8$ is $448x^6$.

Determine the value of a , $a > 0$.

$$\begin{aligned} x &\rightarrow x & t_{k+1} &= {}_8 C_k x^{8-k} a^k \\ y &\rightarrow a & & \\ k &\rightarrow k & & \\ n &\rightarrow 8 & & \end{aligned}$$

term in x^6 , $k=2$

$$\begin{aligned} t_3 &= {}_8 C_2 x^6 a^2 \\ &= 28x^6 a^2 \\ 448x^6 &= 28x^6 a^2 \\ \frac{448x^6}{28x^6} &= a^2 \\ 16 &= a^2 \\ a &= 4 \\ a > 0, a &= \underline{\underline{4}} \end{aligned}$$

b) One term in the expansion of

$(x+b)^{11}$ is $-4455x^8$.

Determine the value of b .

$$\begin{aligned} x &\rightarrow x & t_{k+1} &= {}_{11} C_k x^{11-k} b^k \\ y &\rightarrow b & & \\ k &\rightarrow k & & \\ n &\rightarrow 11 & & \end{aligned}$$

term in x^8 , $k=3$. $11-k=8$ $k=3$

$$\begin{aligned} t_4 &= {}_{11} C_3 x^8 b^3 \\ t_4 &= 165x^8 b^3 \\ -4455x^8 &= 165x^8 b^3 \\ \frac{-4455x^8}{165x^8} &= b^3 \\ -27 &= b^3 \\ b &= \underline{\underline{-3}} \end{aligned}$$

7. Find the indicated term of each expansion.

a) the constant term in the expansion of $(x^2 - \frac{1}{x})^6$

$x \rightarrow x^2$
 $y \rightarrow -\frac{1}{x}$
 $k \rightarrow k$
 $n \rightarrow 6$

$$\begin{aligned}
 t_{k+1} &= {}^6C_k (x^2)^{6-k} \left(-\frac{1}{x}\right)^k \\
 &= {}^6C_k x^{12-2k} (-1)^k (x)^{-k} \\
 &= {}^6C_k (-1)^k x^{12-3k}
 \end{aligned}$$

$12 - 2k + (-k) = 12 - 3k$

constant term $12 - 3k = 0$
 $k = 4$

$$t_5 = {}^6C_4 (-1)^4 x^0 = 15$$

constant term = 15

b) the term independent of x in $(2x + \frac{1}{x^3})^8$

$x \rightarrow 2x$
 $y \rightarrow \frac{1}{x^3}$
 $k \rightarrow k$
 $n \rightarrow 8$

constant

$$\begin{aligned}
 t_{k+1} &= {}^8C_k (2x)^{8-k} \left(\frac{1}{x^3}\right)^k \\
 &= {}^8C_k (2^{8-k}) x^{8-k} x^{-3k} \\
 &= {}^8C_k 2^{8-k} x^{8-4k}
 \end{aligned}$$

$8 - k + (-3k) = 8 - 4k$ constant $k = 0$
 $8 - 4k = 0$
 $k = 2$

$$t_3 = {}^8C_2 2^6 x^0 = 1792$$

term = 1792

Multiple Choice

8. In the expansion of $(x - 3)^8$, the numerical coefficient of the third degree term is

- A. -13 608
- B. 5670
- C. -1512
- D. 252

$x \rightarrow x$
 $y \rightarrow -3$
 $k \rightarrow k$
 $n \rightarrow 8$

$$\begin{aligned}
 t_{k+1} &= {}^8C_k x^{8-k} (-3)^k \\
 \text{term in } x^3 &\rightarrow 8 - k = 3, k = 5 \\
 t_6 &= {}^8C_5 x^3 (-3)^5 \\
 &= 56 x^3 (-243) \\
 &= -13608 x^3
 \end{aligned}$$

9. If the first term in the expansion of $(3x + 2y)^n$ is $243x^5$, then the coefficient of the second term is

- A. 30
- B. 162
- C. 240
- D. 810

$$\begin{aligned}
 t_1 &= {}^nC_0 (3x)^n = 243x^5 \\
 3^n x^n &= 243x^5 \\
 n &= 5 \\
 t_2 &= {}^5C_1 (3x)^4 (2y)^1 \\
 &= 5 (81x^4) (2y) \\
 &= 810 x^4 y
 \end{aligned}$$

Numerical Response

10. If $(2x + 3)^{3n+4}$ has 38 terms, then the value of n is _____.

(Record your answer in the numerical response box from left to right.)

1	1		
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$k+1 = 38 \rightsquigarrow 3n+4+1 = 38$

$3n = 33 \quad n = 11$

11. The term in x^{11} in the expansion of $(x^2 + \frac{1}{x})^{10}$ has a numerical coefficient of _____.

(Record your answer in the numerical response box from left to right.)

1	2	0	
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$x \rightarrow x^2$
 $y \rightarrow \frac{1}{x}$
 $k \rightarrow k$
 $n \rightarrow 10$

$t_{k+1} = {}^{10}C_k (x^2)^{10-k} (\frac{1}{x})^k$
 $= {}^{10}C_k x^{20-2k} x^{-k}$
 $= {}^{10}C_k x^{20-3k}$
 $20-3k = 11 \quad 9 = 3k \quad k = 3$
 $t_4 = {}^{10}C_3 x^{11}$
 $= 120x^{11}$

12. In the expansion of $(3x + 2y)^6$, the numerical coefficient of the term containing x^4y^2 is _____.

(Record your answer in the numerical response box from left to right.)

4	8	6	0
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$x \rightarrow 3x$
 $y \rightarrow 2y$
 $k \rightarrow k$
 $n \rightarrow 6$

$t_{k+1} = {}^6C_k (3x)^{6-k} (2y)^k$
 $= {}^6C_k 3^{6-k} x^{6-k} 2^k y^k$
 term in x^4y^2 , $k=2$
 $t_3 = {}^6C_2 3^4 x^4 2^2 y^2$
 $= 15(81)(4)x^4y^2$
 $= 4860x^4y^2$

13. A term of the binomial expansion $(cx + y)^6$, where c is a positive integer, is $42xy^5$. The value of c is _____.

(Record your answer in the numerical response box from left to right.)

7			
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$x \rightarrow cx$
 $y \rightarrow y$
 $k \rightarrow k$
 $n \rightarrow 6$

$t_{k+1} = {}^6C_k (cx)^{6-k} y^k$
 term in xy^5 , $k=5$
 $t_6 = {}^6C_5 (cx)^1 y^5$
 $= 6cx y^5$
 $42xy^5 = 6cx y^5$
 $\frac{42xy^5}{6xy^5} = c$
 $c = 7$

Answer Key

1. a) $105x^{13}y^2$ b) $924p^6q^6$
 2. $x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$
 3. a) $5ab^4$ b) $-12x^5$ c) $314\ 928\ x^7y^2$ d) $-280a^{11}$ e) $-20x^3$
 4. a) $8x^3 + 12x^2y + 6xy^2 + y^3$ b) $x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$ 5. a) $-1760x^3$ b) $-2835x^4$

6. a) 4 b) -3 7. a) 15 b) 1792 8. A 9. D 10.

1	1		
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 11.

1	2	0	
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 12.

4	8	6	0
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 13.

7			
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