Assignment

1. The Athletic Council decides to form a sub-committee of 6 council members to look at a new sports program. There are a total of 15 Athletic Council members, 6 females and 9 males. How many different ways can the sub-committee consist of at most one male?

0m6F + 1m5F

$({}_9C_0)({}_6C_6) + ({}_9C_1)({}_6C_5) = (1)(1) + 9(6) = 1 + 54 = \underline{55}$

2. A group of 4 journalists is to be chosen to cover a murder trial. There are 5 male and 7 female journalists available. How many possible groups can be formed

a) consisting of 2 men and 2 women?

$({}_5C_2)({}_7C_2)$

$= 10(21)$

$= 210$

b) consisting of at least one woman?

#no restrictions − # no women

${}_{12}C_4 - {}_5C_4$

$= 495 - 5$

$= \underline{490}$

3. Consider a standard deck of 52 cards. How many different four card hands have

a) at least one black card?

#no restrictions − #noblackcards

$= {}_{52}C_4 - {}_{26}C_4$

$= 270725 - 14950$

$= 255775$

b) at least 2 kings?

$2K2\bar{K} + 3K1\bar{K} + 4K$

$= ({}_4C_2)({}_{48}C_2) + ({}_4C_3)({}_{48}C_1) + {}_4C_4$

$= 6(1128) + 4(48) + 1$

$= 6961$

c) two pairs?

$({}_4C_2)({}_4C_2)({}_{13}C_2)$

$= 6(6)(78)$

$= 2808$

d) at most 2 clubs? → 52−13

$0C4\bar{C} + 1C3\bar{C} + 2C2\bar{C}$

$= {}_{39}C_4 + ({}_{13}C_1)({}_{39}C_3) + ({}_{13}C_2)({}_{39}C_2)$

$= 82251 + 13(9139) + 78(741)$

$= 258\,856.$

4. City Council decides to form a sub-committee of five aldermen to investigate transportation concerns. There are 4 males and 7 females. How many different ways can the sub-committee be formed consisting of at least one female member?

#no restrictions − # no females

$= {}_{11}C_5 - 0$ ← b/c there's only 4 men so there will always be 1 female on committee

$= \underline{462}$

5. An all-night showing at a movie theatre is to consist of five movies. There are fourteen different movies available, ten disaster movies and four horror movies. How many possib schedules include:

a) at least one horror movie?

\#(no restrictions) - #(no horrors)

$= {}_{14}C_5 - {}_{10}C_5$

$= 2002 - 252$

$= \underline{1750}$

b) at least four disaster movies?

4D 1H or 5D

$= ({}_{10}C_4)({}_4C_1) + {}_{10}C_5$

$= 210(4) + 252$

$= \underline{1092}$

c) both "Airport Disaster" and "Halloween Horror"?

choose 3 from remaining 12.

${}_{12}C_3 = \underline{\underline{220}}$

6. Use "guess and check" on a calculator to determine the solution(s) to the following equations.

a) $\binom{n}{2} = 105$ $n = 15$

b) ${}_nC_3 = 84$ $n = 9$

c) ${}_{11}C_n = 330$ $n = 4, 7$.

7. Algebraically determine the solution to the equation ${}_nC_7 = {}_{n+1}C_8$.

$$\frac{n!}{(n-7)!\,7!} = \frac{(n+1)!}{((n+1)-8)!\,8!}$$

8. Determine the number of diagonals in

a) a regular hexagon

${}_6C_2 - 6 = 15 - 6 = 9$

b) a regular decagon

${}_{10}C_2 - 10 = 45 - 10 = 35$

9. Show that the number of diagonals in a regular p-sided polygon is $\frac{p(p-3)}{2}$.

${}_pC_2 - p$

$= \frac{p!}{(p-2)!\,2!} - p$

$= \frac{p(p-1)(p-2)!}{(p-2)!(2)} - p$

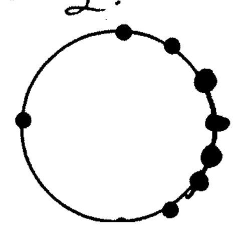
$\frac{p(p-1)}{2} - p$

$= \frac{p^2 - p - 2p}{2}$

$= \frac{p^2 - 3p}{2} = \frac{p(p-3)}{2}$.

10. There are eight visible points on the circle below. How many triangles can be made using these eight points?

3



${}_8C_3 = 56$ triangles.

Multiple Choice

11. After everyone had shaken hands once with everyone else in a room, there was a total of 66 handshakes. How many people were in the room?

A. 11
B. 12
C. 33
D. 67

$_nC_2 = 66$

$\frac{n!}{(n-2)!\,2!} = 66$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2} = 66$

$n^2 - n = 132$

$n^2 - n - 132 = 0$

$(n-12)(n+11) = 0$

$n = 12, -11$ reject.

12. There are 20 different ways of selecting three students from a class of students. Which the following equations can be used to determine the number, n, of students in the class?

A. $n^3 - 3n^2 - 2n - 20 = 0$
B. $n^3 - 3n^2 - 2n - 120 = 0$
C. $n^3 - 3n^2 + 2n - 20 = 0$
D. $n^3 - 3n^2 + 2n - 120 = 0$

$_nC_3 = 20$

$\frac{n!}{(n-3)!\,3!} = 20$

$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}(6)} = 20$

$n(n-1)(n-2) = 120$

$n(n^2 - 3n + 2) - 120 = 0$

$n^3 - 3n^2 + 2n - 120 = 0$

Numerical Response

13. The number of ways that a selection of 7 students can be chosen from a class of 28 is the same as the number of ways that n students can be chosen from the same class. The value of n is _____ .

(Record your answer in the numerical response box from left to right.)

2	1		

$_{28}C_7 = {}_{28}C_{21} \Rightarrow n = 21$

14. Collinear points are points which share the same straight line. The number of triangles which can be formed from 10 points if no three of the points are collinear is _____ .

(Record your answer in the numerical response box from left to right.)

1	2	0	

$_{10}C_3 = 120$

15. There are 170 diagonals in a polygon. The number of sides of the polygon is _____ .

└ 2 pts.

(Record your answer in the numerical response box from left to right.)

2	0		

$_nC_2 - n = 170$

$\frac{n!}{(n-2)!\,2!} - n = 170$

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}(2)} - n = 170$

$n(n-1) - 2n = 340$

$n^2 - n - 2n - 340 = 0$

$n^2 - 3n - 340 = 0$

$(n-20)(n+17) = 0$

$n = 20, \cancel{-17}$

Answer Key

1. 55
2. a) 210 b) 490
3. a) 255 775 b) 6961 c) 2808 d) 258 856
4. 462
5. a) 1750 b) 1092 c) 220
6. a) 15 b) 9 c) 4 or 7
7. 7
8. a) 9 b) 35
10. 56
11. B
12. D

13.

2	1		

14.

1	2	0	

15.

2	0		