Defining 0!

If we replace r by n in the previous formula we get the number of permutations of n elements taken n at a time. This we know is $n!$.

$${}_nP_n = n! = \frac{n!}{(n-n)!} = \frac{n!}{0!}$$

For this to be equal to $n!$ the value of $0!$ must be 1.

0! is defined to have a value of 1.

Class Ex. #6

In a region, vehicle license plates consist of 2 different letters followed by 4 different digits. If the letters I, O, Y, and Z are not used, determine how many different license plates are possible by

a) the fundamental counting principle

$\underline{22} \cdot \underline{21} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = 2328480$

b) permutations

$({}_{22}P_2)({}_{10}P_4) = 2328480$

Class Ex. #7

Algebraically solve the equation ${}_{n-1}P_2 = 90$.

$${}_{n-1}P_2 = \frac{(n-1)!}{((n-1)-2)!} = \frac{(n-1)!}{(n-3)!} = \frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = (n-1)(n-2)$$

$${}_{n-1}P_2 = 90 \Rightarrow (n-1)(n-2) = 90 \Rightarrow n^2 - 3n + 2 = 90 \Rightarrow n^2 - 3n - 88 = 0$$

$(n-11)(n+8) = 0$ $n = 11$ + -8 reject

$\boxed{n = 11}$

Note In many cases involving simple permutations, the fundamental counting principle can be used in place of the permutation formulas. new

Complete Assignment Questions #7 - #15

Assignment

1. Without using a calculator, determine the value of

a) $5!$

$5 \times 4 \times 3 \times 2 \times 1$

$= 120$

b) $\frac{10!}{8!}$

$= \frac{10 \times 9 \times 8!}{8!}$

$= 90$

c) $\frac{99!}{100!} = \frac{\cancel{99!}}{100 \cdot \cancel{99!}} = \frac{1}{100}$

2. Express as single factorials.

a) $6 \times 5 \times 4 \times 3 \times 2 \times 1$ **b)** $9 \times 8 \times 7 \times 6!$ **c)** $(n+2)(n+1)n(n-1) \ldots \times 3 \times 2 \times$

$6!$ $9!$ $(n+2)!$

3. Express as a quotient of factorials.

a) $9 \times 8 \times 7 \times 6$ **b)** $20 \times 19 \times 18$ **c)** $(n+2)(n+1)n$

$\frac{9!}{5!}$ $\frac{20!}{17!}$ $\frac{(n+2)!}{(n-1)!}$

4. Use a calculator to determine the exact value of the following:

a) $10!$ **b)** $\frac{8!}{4!}$ **c)** $\frac{15!}{10!\,5!}$ **d)** $\left(\frac{25!}{21!}\right)\left(\frac{7!}{11!}\right)$ $\frac{115}{3}$

3 628 800 1680 3003

5. Simplify the following expressions. Leave the answer in product form where appropriate.

a) $\frac{n!}{n}$ **b)** $\frac{(n-3)!}{(n-2)!}$ **c)** $\frac{(n+1)!}{(n-1)!}$ **d)** $\frac{(3n)!}{(3n-2)!}$

$\frac{n(n-1)!}{n}$

$=(n-1)!$

$\frac{(n-3)!}{(n-2)(n-3)!}$

$=\frac{1}{n-2}$

$\frac{(n+1)(n)(n-1)!}{(n-1)!}$

$=n(n+1)$

$\frac{3n(3n-1)(3n-2)!}{(3n-2)!}$

$=3n(3n-1)$

6. Solve the equation.

a) $\frac{(n+1)!}{n!} = 6$ **b)** $(n+1)! = 6(n-1)!$

$\frac{(n+1)n!}{n!} = 6$

$n+1 = 6$

$n = 5$

$\frac{(n+1)(n)(n-1)!}{\cancel{6}(n-1)!} = 6$

$n(n+1) = 6 \rightarrow n^2+n-6=0$

$(n+3)(n-2)=0$

$\cancel{n=-3}, 2$ $\boxed{n=2}$

c) $\frac{(n+2)!}{n!} = 12$ **d)** $\frac{(n+1)!}{(n-2)!} = 20(n-1)$

$\frac{(n+2)(n+1)n!}{n!} = 12$

$n^2+3n+2=12$

$n^2+3n-10=0$

$(n+5)(n-2)=0$

$n=-5, 2$ $\boxed{n=2}$

reject

$\frac{(n+1)(n)\cancel{(n-1)}\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20\cancel{(n-1)}$

$n(n+1) = 20$

$n^2+n-20=0$

$(n+5)(n-4)=20$

$n=-5, 4$ $\boxed{n=4}$

reject

7. Determine the number of arrangements that can be made using all of the letters in the worc

a) DOG **b)** DUCK **c)** SANDWICH **d)** CANMORE

$3! = 6$ $4! = 24$ $8! = 40320$ $7! = 5040$

8. Consider the number of five-digit numbers that can be made from the digits 2, 3, 4, 7, and if no digit can be repeated. Express your answer using

a) factorial notation **b)** $_nP_r$ notation **c)** the fundamental counting principle

$5!$ $_5P_5$ $\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = \underline{120}$

9. **a)** Use the formula for $_nP_r$ to show that $_7P_0 = 1$.

$$_7P_0 = \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$$

b) Explain why n must be greater than or equal to r in the notation $_nP_r$.

You cannot arrange more elements than the number of elements there are to begin with. Also $(n-r)!$ would lead to the factorial of a ⊖ number.

10. In each case determine the number of arrangements of the given letters by
i) using the fundamental counting principle **ii)** writing in $_nP_r$ form and evaluating

a) two letters from the word **GOLDEN**

$6 \cdot 5 = 30$

$_6P_2 = 30$

b) three letters from the word **CHAPTER**

$8 \cdot 7 \cdot 6 = 336$

$_8P_3 = 336$

c) four letters from the word **WEALTH**

$6 \cdot 5 \cdot 4 \cdot 3 = 360$

$_6P_4 = 360$

d) one letter from the word **VALUE**

5

$_5P_1 = 5$

11. Solve each equation, where n is an integer.

a) $\frac{n!}{84} = {}_{n-2}P_{n-4}$

$$\frac{n!}{84} = \frac{(n-2)!}{[(n-2)-(n-4)]!}$$

$$\frac{n!}{84} = \frac{(n-2)!}{2!}$$

$$\frac{n!}{(n-2)!} = \frac{84}{2!} \qquad \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$$

$$n^2 - n - 42 = 0$$

$$(n-7)(n+6) = 0$$

$n = 7$, -6 reject $\boxed{n = 7}$

b) $_nP_4 = 8(_{n-1}P_3)$

$$\frac{n!}{(n-4)!} = \frac{8(n-1)!}{[(n-1)-3]!}$$

$$\frac{n!}{(n-4)!} = \frac{8(n-1)!}{(n-4)!}$$

$$n! = 8(n-1)!$$

$$\frac{n!}{(n-1)!} = 8$$

$$\frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = 8$$

$\boxed{n = 8}$

12. How many numbers (up to a maximum of four digit numbers) can be made from the digits 2, 3, 4, and 5 if no digit can be repeated?

$_4P_1 + {_4P_2} + {_4P_3} + {_4P_4}$

$= 4 + 12 + 24 + 24$

$= 64$

Multiple Choice

13. In a ten-team basketball league, each team plays every other team twice, once at home and once away. The number of games that are scheduled is

order is important

$_{10}P_2 = 90$

A. 45 **B.** 90
C. 100 **D.** 180

14. The value of $_nP_2$ is

$\frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!}$

$= n(n-1)$

A. $\frac{n}{n-2}$ **B.** $\frac{n!}{2!}$

C. $\frac{n}{2}$ **D.** $n(n-1)$

Numerical Response

15. In a competition on the back of a cereal packet, seven desirable qualities for a kitchen (eg spaciousness, versatility, etc.) must be put in order of importance. The number of different entries that must be completed in order to ensure a winning order is ______ .

$7! = 5040$

(Record your answer in the numerical response box from left to right.)

5	0	4	0

Answer Key

1. a) 120 b) 90 c) $\frac{1}{100}$
2. a) 6! b) 9! c) $(n + 2)!$
3. a) $\frac{9!}{5!}$ b) $\frac{20!}{17!}$ c) $\frac{(n+2)!}{(n-1)!}$
4. a) 3 628 800 b) 1680 c) 3003 d) $\frac{115}{3}$
5. a) $(n - 1)!$ b) $\frac{1}{n-2}$ c) $n(n + 1)$ d) $3n(3n - 1)$
6. a) $n = 5$ b) $n = 2$ c) $n = 2$ d) $n = 4$
7. a) 6 b) 24 c) 40 320 d) 5040
8. a) 5! b) $_5P_3$ c) $5 \times 4 \times 3 \times 2 \times 1 = 120$
9. a) $_7P_0 = \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$
   b) You cannot arrange more elements than the number of elements there are to begin with.
10. a) $_6P_2 = 30$ b) $_8P_3 = 336$ c) $_6P_4 = 360$ d) $_5P_1 = 5$
11. a) $n = 7$ b) $n = 8$ 12. 64 13. B 14. D 15.

5	0	4	0