Trigonometry - Functions and Graphs Lesson #11: Practice Test

Section A ***No calculator may be used for this section of the test.***

1. If the value of csc X is positive and the value of cot X is negative, then

   A. $0 < X < \frac{\pi}{2}$ **B.** $\frac{\pi}{2} < X < \pi$ C. $\pi < X < \frac{3\pi}{2}$ D. $\frac{3\pi}{2} < X < 2\pi$

   sinX is (+) & tanX is (−) in Q2

2. The exact value of $\sec \frac{7\pi}{6}$ is

   A. $\frac{2\sqrt{3}}{3}$ **B.** $-\frac{2\sqrt{3}}{3}$ C. 2 D. −2

   ref∠ $= \frac{\pi}{6}$

   $\sec \frac{7\pi}{6} = -\sec \frac{\pi}{6} = \frac{-1}{\cos \frac{\pi}{6}} = \frac{-1}{\sqrt{3}/2} = \frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

S	A
T	C

Use the following information to answer the next question.

As part of a review for an exam, a student makes four statements in which the answers are either quadrant 1, quadrant 2, quadrant 3, or quadrant 4.

Statement 1: A rotation angle of $\frac{65\pi}{6}$ radians in standard position terminates in quadrant a.

Statement 2: If $\tan A < 0$ and $\sec A > 0$, then angle A terminates in quadrant b.

Statement 3: Angles in standard position coterminal with an angle of $-\frac{7\pi}{4}$ radians are all in quadrant c.

Statement 4: For $0 \le x \le 2\pi$, the roots of the equation $\csc x = 1.5$ are in quadrant 1 and quadrant d.

Numerical Response

1. Write the value of a in the first box, the value of b in the second box, the value of c in the third box, and the value of d in the last box.

   (Record your answer in the numerical response box from left to right.)

2	4	1	2

$\frac{65\pi}{6} - 3(2\pi) = \frac{5\pi}{6}$ Q2 ② tanA < 0, cosA > 0 Q4 ③ Q1 ④ cscx is (+) son sinX is (+) Q1+2.

3. The period of the graph of the function $f(x) = \tan 2\pi x$ is

   A. 0.5 B. 1 C. π D. 2π

   period $= \frac{\pi}{2\pi} = \frac{1}{2} = 0.5$

4. Compared to the graph of $y = \cos x$, the horizontal phase shift of $y = \cos\left(\frac{1}{4}x + \frac{\pi}{2}\right)$ is

A. $\frac{\pi}{8}$ to the left B. $\frac{\pi}{2}$ to the left

(C.) 2π to the left D. 8π to the left

$y = \cos\frac{1}{4}(x + 2\pi)$

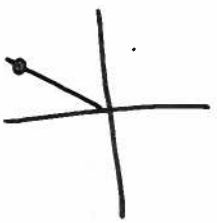
h.p.s $= 2\pi$ (L)

5. The terminal arm of angle x, in standard position, contains the ordered pair $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$

A possible value for angle x is

A. $\frac{4\pi}{3}$ B. $\frac{7\pi}{6}$

C. $\frac{-4\pi}{3}$ (D.) $\frac{-7\pi}{6}$



ref $\angle = \frac{\pi}{6}$

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$

Section B ***A graphing calculator may be used for the remainder of the test.***

6. An angle has a degree measure of $2\pi°$. To the nearest tenth of a radian, the angle has a radian measure of

(A.) 0.1 B. 2

C. 19.7 D. 360

$2\pi \cdot \frac{\pi}{180} = 0.109$

7. The minimum value of the function $f(x) = a\cos bx - d$, where $a > 0$, is

(A) $-a - d$ B. $a - d$

C. $a - b - d$ D. $-b - d$

$a(-1) - d = -a - d$.

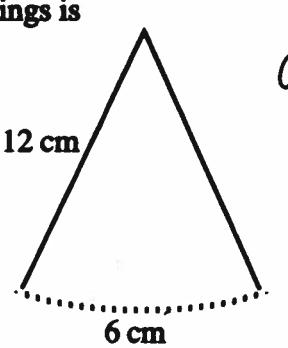
8. A pendulum 12 cm long swings through an arc of length 6 cm. To the nearest degree, the angle through which the pendulum swings is

A. 115°

B. 90°

(C.) 29°

D. 1°



$a = r\theta$

$\theta = \frac{a}{r}$

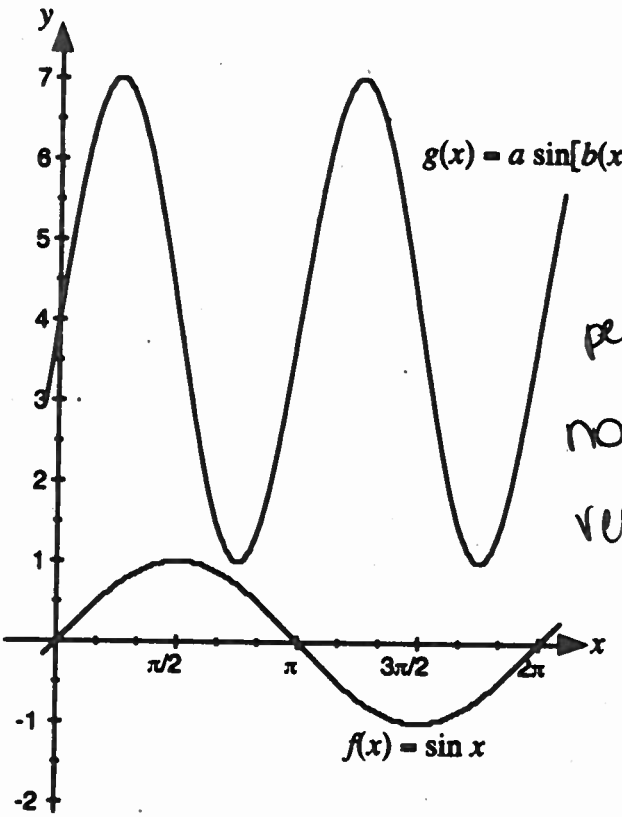
$= \frac{6}{12}$

$= 0.5$ rads

$0.5 \times \frac{180}{\pi} = 28.64°$

Use the following information to answer the next question.

The partial graphs of $f(x) = \sin x$ and $g(x) = a\sin[b(x-c)] + d$, where a, b, c and d are whole numbers, are shown below.



amp $= \frac{7-1}{2} = 3 = a$

period $= \pi$ $b = \frac{2\pi}{\pi} = 2$

no h.p.s $c = 0$

vert. dis $= \frac{7+1}{2} = 4$ $d = 4$

Numerical Response

2. Write the value of a in the first box, the value of b in the second box, the value of c in the third box, and the value of d in the last box.

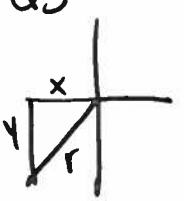
(Record your answer in the numerical response box from left to right.)

3	2	0	4

$a = 3, b = 2, c = 0, d = 4$

9. If $\tan\theta = \frac{7}{24}$ and $\cos\theta < 0$, then the exact value of $\sin\theta$ is

Q3



$\tan\theta = \frac{7}{24} = \frac{y}{x}$

let $x = -24, y = -7$

$r^2 = (-24)^2 + (-7)^2 = 625$

$r = 25$

$\sin\theta = \frac{y}{r} = \frac{-7}{25}$

A. -24 B. $-\frac{24}{25}$

(C.) $-\frac{7}{25}$ D. -7

10. The y-intercept of the graph of $y = \cos bx + d$ is

let $x = 0$

$y = \cos 0 + d$

$= 1 + d$

A. b B. $b + 1$

C. d (D.) $d + 1$

Numerical Response

3. If $\tan x = -\frac{2}{5}$, $\pi < x < 2\pi$, then the measure of x, to the nearest 0.01 radians, is ____.

(Record your answer in the numerical response box from left to right.)

5	.	9	0

S | A
T | C

ref angle $(\tan^{-1}\frac{2}{5}) = 0.3805$

in Q4 $x = 2\pi - 0.3805 = 5.9026$

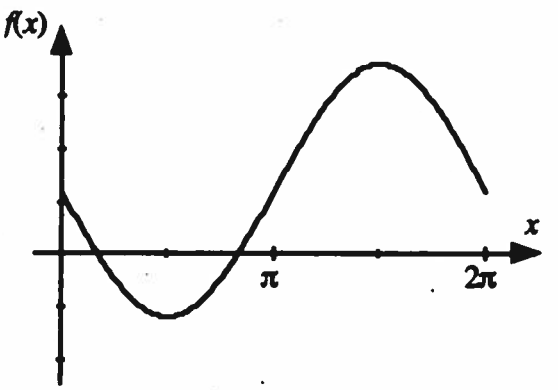
11. The point $A\left(\frac{3\pi}{2}, -2\right)$ lies on the graph of $y = k\sin\left(x - \frac{\pi}{3}\right)$. The value of k is

(A.) 4 **B.** $\frac{4\sqrt{3}}{3}$ $-2 = k\sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right)$ $-2 = k\left(-\frac{1}{2}\right)$

C. -4 **D.** 1 $-2 = k\sin\left(\frac{7\pi}{6}\right)$ $k = \frac{-2}{-1/2} = 4$

Use the following information to answer the next question.

The partial graph of $f(x) = a\cos[b(x-c)] + d$ where $a > 0$, is shown.



The minimum point on the graph is $S\left(\frac{\pi}{2}, -2.4\right)$ and the maximum point is $T\left(\frac{3\pi}{2}, 7.2\right)$.

12. The value of a, to the nearest tenth, is

A. 2.4 (B.) 4.8 $\text{amp} = \frac{7.2 - (-2.4)}{2} = 4.8$

C. 6.3 **D.** 9.6 $a = 4.8$

13. Which of the following functions have the same period?

$f(x) = 2\sin[4(x-\pi)] + 1$ $g(x) = 2\cos[(x-\pi)] + 5$

$h(x) = 3\tan[2(x-\pi)]$ $k(x) = \sin[2(x-\pi)] + 5$

A. $f(x)$ and $g(x)$ (B.) $f(x)$ and $h(x)$

C. $h(x)$ and $k(x)$ **D.** $g(x)$ and $k(x)$

function	f	g	h	k
b-value	4	1	2	2
period	$\frac{2\pi}{4} = \frac{\pi}{2}$	2π	$\frac{\pi}{2}$	$\frac{2\pi}{2} = \pi$

14. Which of the following statements is true for all values of x for which the function is defined?

A. $\sin(-x) = \sin x$ **B.** $\cos(-x) = \cos x$ **C.** $\tan(-x) = \tan x$ **D.** $\cot(-x) = \cot x$

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$ $\cot(-x) = -\cot(x)$

15. A fan blade has a diameter of 32 cm. If the blade is rotated 532° about its centre, then a point on the tip of a blade would travel a distance of

A. 1.49 m

B. 2.97 m

C. 85.1 m

D. 170.2 m

$532° = 532 \times \frac{\pi}{180} = \frac{133\pi}{45}$ rads

$r = 16$ cm

$a = r\theta = 16\left(\frac{133\pi}{45}\right) = 148.50$ cm

$= 1.49$ m

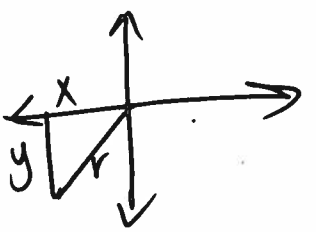
16. If $\sin\theta = -0.40$ and $\tan\theta > 0$, then $\sec\theta$, to the nearest hundredth, is

A. 0.92

B. –0.92

C. –1.09

D. 1.09



Q3.

$\sin\theta = -0.40 = \frac{y}{r}$

let $y = -0.40$, $r = 1$

$x^2 + (-0.4)^2 = 1^2$

$x = (-0.92)$ in Q3.

$\sec\theta = \frac{r}{x} = \frac{1}{-0.92} = -1.09$

Numerical Response

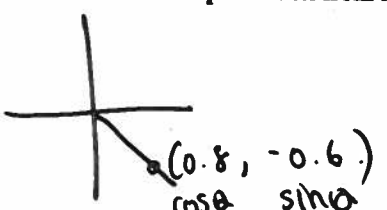
4. A point on the unit circle is approximated by the ordered pair (0.80, –0.60). To the nearest degree, the angle in standard position between 0° and 360° with a terminal arm passing through this point is _____.

(Record your answer in the numerical response box from left to right.)

3	2	3	

Q4

$\cos\theta = 0.80$



ref $\angle = 37°$

rot. angle $= 360 - 37$

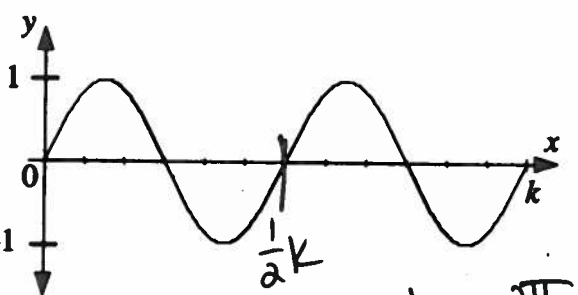
$= 323°$

17. The partial graph shown is in the domain $0 \le x \le k$.

A possible equation of the graph is

A. $y = \sin\left(\frac{4\pi}{k}\right)x$ **B.** $y = \sin\left(\frac{2\pi}{k}\right)x$

C. $y = \sin\left(\frac{\pi}{k}\right)x$ **D.** $y = \sin\left(\frac{k}{2}\right)x$



period $= \frac{1}{2}k$ $b = \frac{2\pi}{\frac{1}{2}k} = \frac{4\pi}{k}$.

Numerical Response

5. In the diagram, an arc of length 8 cm subtends an angle of 90° at the centre of the circle To the nearest tenth of a cm, the diameter of the circle is

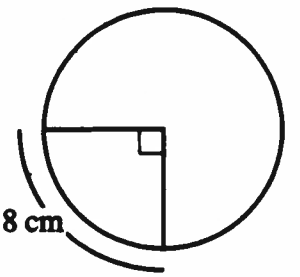
$a = 8\text{cm} \quad \theta = 90° = \frac{\pi}{2}$ rads

$a = r\theta$

$r = \frac{a}{\theta} = \frac{8}{\pi/2} = \frac{16}{\pi}$

$\text{diameter} = 2\left(\frac{16}{\pi}\right) = 10.1859$

$= 10.2\text{cm}$



(Record your answer in the numerical response box from left to right.)

1	0	.	2

18. The function $f(\theta) = \tan\theta$ is stretched horizontally by a factor of $\frac{1}{2}$ about the y-axis.

The asymptotes of the graph of the transformed function can be described by which of th following where n is an integer?

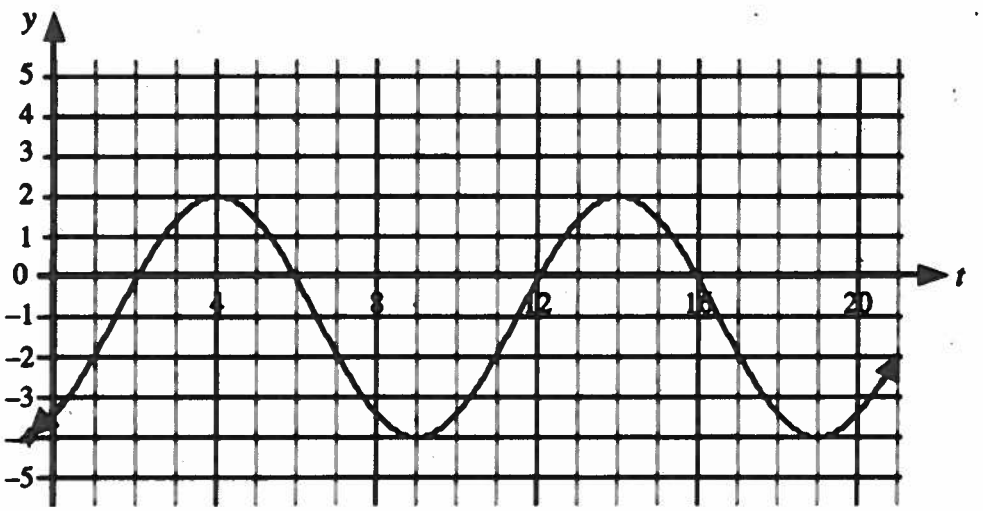
asymptotes of $\tan\theta$ are $\frac{\pi}{2} + n\pi, n \in$
-after horizontal stretch by a factor of 1/2 asymptotes are $\frac{1}{2}\left(\frac{\pi}{2} + n\pi\right) = \frac{\pi}{4} + \frac{n\pi}{2}, n \in$

A. $\theta = 2n\pi$ B. $\theta = \frac{n\pi}{2}$

C. $\theta = \pi + 2n\pi$ D. $\frac{\pi}{4} + \frac{n\pi}{2}$

Numerical Response

6. The graph shown is a partial graph of $y = a \sin b(t - c) + d$ where $a, b > 0$.



The value of $a + b$, to the nearest tenth, is ____.

(Record your answer in the numerical response box from left to right.)

3	.	6	

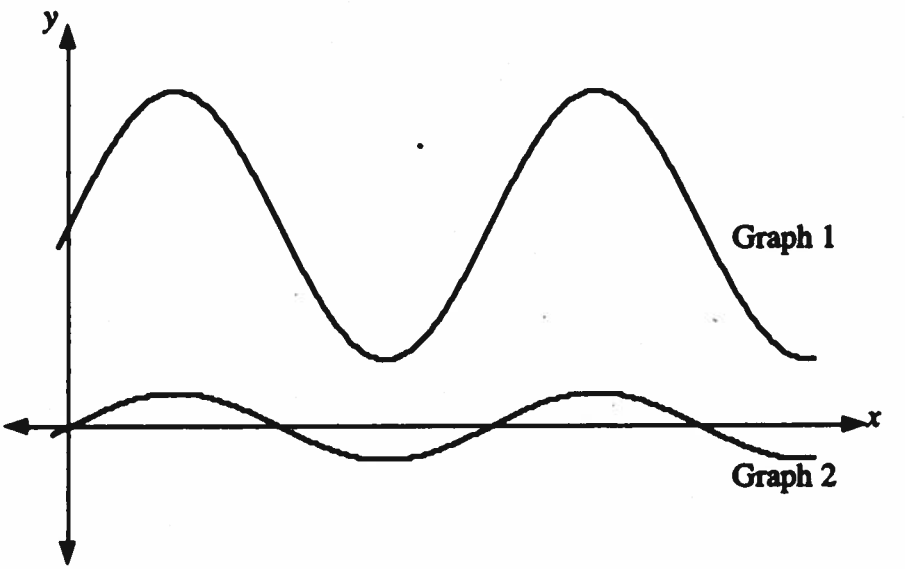
$\text{amp} = \frac{2 - (-4)}{2} = 3 \quad a = 3$

$\text{period} = 14 - 4 = 10 \quad b = \frac{2\pi}{10} = \frac{\pi}{5}$

$a + b = 3 + \frac{\pi}{5} = 3.628$

Use the following information to answer the next question.

The partial graphs of two trigonometric functions are shown.



Graph 1 has equation $y = a\cos[b(x-c)] + d$. The equation for graph 2 can be obtained from the equation for graph 1 by changing two of the parameters a, b, c, and d.

19. The two parameters that are changed are

A. a and d **B.** b and d **C.** a and c **D.** a and b

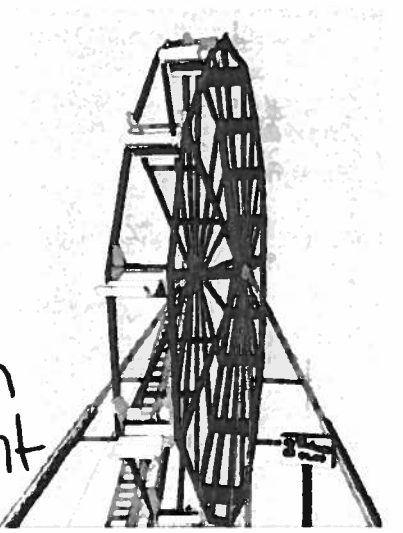
amp + vertical displacement are changed.

20. The height, h metres above the ground, at time, t, of a student on the circular Ferris Wheel shown is given by the equation $h(t) = a\cos b(t-c) + d$.

The diameter of the wheel is 12 metres and the student starts at the bottom of the Ferris Wheel at $t = 0$ at a height of 3 metres above the ground.

The value of d in the equation is

A. 15
B. 12
C. 9
D. 6.



min height = 3m
max height = 3+12 = 15m
d = vert. displacement
$= \frac{max + min}{2} = \frac{15+3}{2} = 9.$

Written Response

On one particular day in Kylebane harbour, the equation of the sinusoidal function that represents the relationship between the depth of water, y metres, and the time, t hours after midnight, is $y = 4\sin\left(\frac{\pi}{6}(t+2)\right) + 15$.

- Calculate the depth of water 4 hours after midnight.

$y = 4\sin\left(\frac{\pi}{6}(4+2)\right)+15 = 4\sin\pi + 15 = 0+15 = 15$ m.

- State the maximum and minimum depths of water in the harbour.

max $= 4(1)+15 = 19$ m min $= 4(-1)+15 = 11$ m.

- Calculate, as an **exact value** and as a **decimal** to the nearest tenth of a metre, the depth of water at midnight.

$t=0$ $y = 4\sin\left(\frac{\pi}{6}(0+2)\right)+15 = 4\sin\frac{\pi}{3}+15 = 4\left(\frac{\sqrt{3}}{2}\right)+15$

$= 2\sqrt{3}+15$,

$= 18.5$ m

- A large supertanker is in the harbour at midnight. It can remain in the harbour if the depth of water in the harbour is at least 12 metres. Since the minimum depth of water in the harbour is less than 12 metres, the tanker will need to leave the harbour before the depth falls below 12 metres.

Explain clearly how to use a graphical approach to determine the latest time, to the whole hour, that the supertanker can remain in the harbour. State an appropriate graphing calculator window.

- graph in radian mode

$y_1 = 4\sin\left(\frac{\pi}{6}(x+2)\right)+15$

$y_2 = 12$

} use intersect feature to determine x-int of first point of intersection window x: [0, 24, 4] y: [0, 20, 4]

- Calculate the latest time, to the whole hour, that the supertanker can remain in the harbour.

Answer Key

Multiple Choice

1. B	2. B	3. A	4. C	5. D	6. A	7. A	8. C
9. C	10. D	11. A	12. B	13. B	14. B	15. A	16. C
17. A	18. D	19. A	20. C				

Numerical Response

1. 2 4 1 2
2. 3 2 0 4
3. 5 . 9 0
4. 3 2 3
5. 1 0 . 2
6. 3 . 6

Written Response

- 15 m
- 19 m, 11 m
- $2\sqrt{3}+15$ m, 18.5 m
- graph $Y_1 = 4\sin\left(\frac{\pi}{6}(x+2)\right)+15$, graph $Y_2 = 12$ and use intersect feature of the calculator to determine the x co-ordinate of the first point of intersection of the two graphs. Window may vary, x:[0, 24, 4] y:[0, 20, 4]
- 5 am