| Verifying that Functions are Inverses of Each Other |

Class Ex. #5 Determine $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$ for Class Ex. #2a), where $f(x) = 2x - 3$. What do you notice?

| Two Functions f and g are Inverses of each other if $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ |

| Complete Assignment Questions #5 - #14 |

Assignment

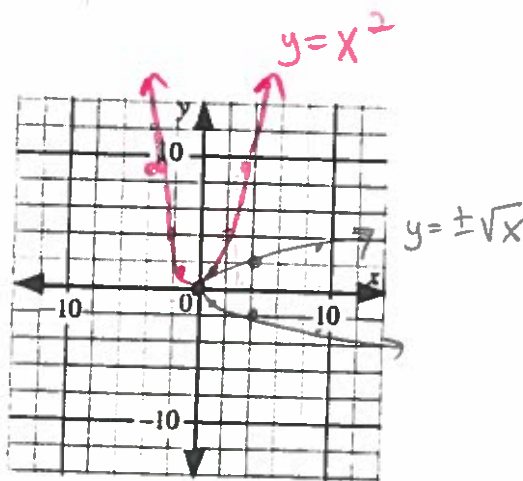1. Consider the relation $y^2 = x$ whose graph is a parabola.

   a) Write the relation in terms of y and sketch the graph of the relation on the grid.

   $$y = \pm\sqrt{x}$$

   b) Determine the equation of the inverse of the relation and sketch the graph of the inverse on the grid.

   $$x^2 = \pm\sqrt{y}^2$$
   $$y = x^2$$

$y = x^2$

$y = \pm\sqrt{x}$

   c) State the domain and range of the relation and the domain and range of the inverse. Does this agree with the second and third bullets in the review at the beginning of the lesson?

   $y = \pm\sqrt{x}$ $y = x^2$ yes → domain of original

   $x | x \geq 0$ $x \in R$ is range of inverse

   $y | y \in R$ $y \geq 0$

2. A parabolic relation is shown in the diagram. The graph passes through the points $(1, -1)$ and $(-2, 1)$, and has a horizontal line of symmetry.

   a) State the coordinates of one other point which lies on the parabola. $(-2, -3)$

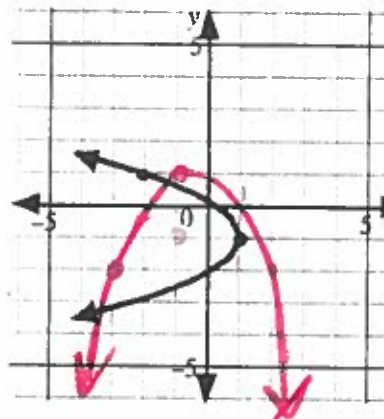   b) State the domain and range of the relation.

   domain $(-\infty, 1]$
   range $(-\infty, \infty)$

   c) State the domain and range of the inverse of the relation.

   domain $(-\infty, \infty)$
   range $(-\infty, 1]$

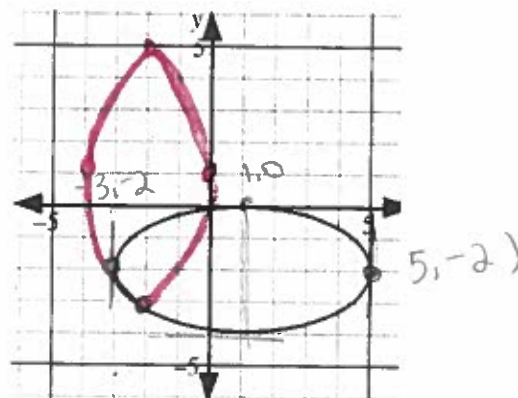   d) Without using a calculator, sketch the inverse of the relation on the grid.

3. The diagram shows the relation with equation $\dfrac{(x-1)^2}{16} + \dfrac{(y+2)^2}{4} = 1$.

   a) State the domain and range of the relation.

   domain $[-3, 5]$ range $[-4, 0]$

   b) State the domain and range of the inverse of the relation.

   domain $[-4, 0]$ range $[-3, 5]$

   c) Without using a calculator, sketch the inverse of the relation on the grid.

   d) Write the inverse of the relation in the form $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, and solve for y.

$$\cdot 6 \left(\frac{(y-1)^2}{16} + \frac{(x+2)^2}{4} = 1 \right)$$

$$(y-1)^2 + 4(x+2)^2 = 16$$

$$(y-1)^2 = -4(x+2)^2 + 16$$

$$y - 1 = \sqrt{-4(x+2)^2 + 16}$$

$$y = \sqrt{16 - 4(x+2)^2} + 1$$
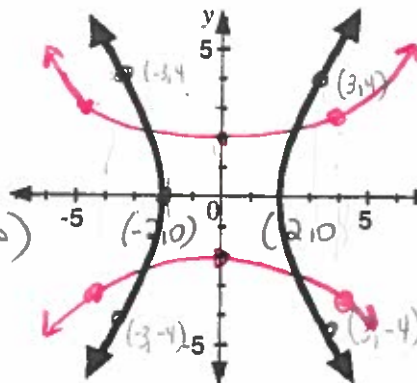
$$= \sqrt{4[4 - (x+2)^2]} + 1$$

FUN!

   e) Use a graphing calculator to verify your sketch in c).

$$y = 2\sqrt{(4 - (x+2)^2)} + 1$$

4. The diagram shows the relation with equation

$\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$. The points $(\pm 2, 0)$, $\left(\pm\sqrt{8}, \pm 3\right)$, and

$\left(\pm 4, \pm\sqrt{27}\right)$ lie on the graph of the relation.

a) State the domain and range of the relation.

domain: $x \mid x \le -2, x \ge 2$ or $(-\infty, -2] \cup [2, \infty)$

$y \mid y \in R$ or $(-\infty, \infty)$

b) State the domain and range of the inverse of the relation.

domain $(-\infty, \infty)$

range $(-\infty, -2] \cup [2, \infty)$

c) Without using a calculator, sketch the inverse of the relation on the grid.

d) Write the inverse of the relation in terms of y.

$36\left(\dfrac{y^2}{4} - \dfrac{x^2}{9} = 1\right)$

$9y^2 - 4x^2 = 36$

$\dfrac{9y^2}{9} = \dfrac{4x^2 + 36}{9}$

$y = \pm\sqrt{\dfrac{4x^2 + 36}{9}} \quad = \pm\sqrt{\dfrac{4x^2}{9} + \dfrac{36}{9}} = \pm\sqrt{\dfrac{4}{9}\left(x^2 + 9\right)}$

$= \pm\dfrac{2}{3}\sqrt{x^2 + 9}$

e) Use a graphing calculator to verify your sketch in c).

5. Use function notation to write the inverse of the following functions.

a) $f(x) = 4x + 5$ **b)** $g(x) = \dfrac{3x - 1}{7}$ **c)** $h(x) = x^3 - 1$

$x = 4y + 5$

$x - 5 = 4y \quad y = \dfrac{x}{4} - \dfrac{5}{4}$

$f^{-1}(x) = \dfrac{1}{4}x - \dfrac{5}{4}$

$x = \dfrac{3y - 1}{7}$

$7x = 3y - 1$

$7x + 1 = 3y$

$g^{-1}(x) = \dfrac{7}{3}x + \dfrac{1}{3}$

$x = y^3 - 1$

$x + 1 = y^3$

$y = \sqrt[3]{x + 1}$

$h^{-1}(x) = \sqrt[3]{x + 1}$

6. **a)** Graph the function $f(x) = x^2 + 4$.

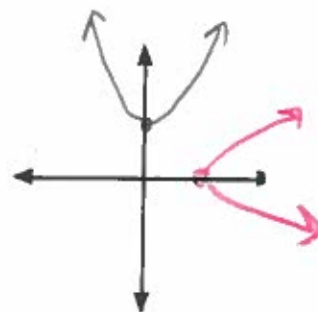**b)** Graph the inverse of $f(x)$.

c) Find the equation of the inverse function in the form $x = f(y)$ and solve for y.

$x = y^2 + 4$

$x - 4 = y^2$

$y = \pm\sqrt{x - 4}$

7. For each of the following functions
   - determine the inverse using the notation $f^{-1}(x)$, where appropriate
   - state the domain and range of the inverse

hint → find domain + range of original + swap it.

a) $f(x) = \sqrt{x+2}$ $x \geq -2$
   $y \geq 0$

$x = \sqrt{y+2}$

$x^2 = y+2$

$y = x^2 - 2$

$f^{-1}(x) = x^2 - 2$

$x \geq 0$
$y \geq -2$

b) $f(x) = (x-2)^2$ $x \in R$
   $y \geq 0$

$x = (y-2)^2$

$\pm\sqrt{x} = y-2$

$y = 2 \pm \sqrt{x}$ (not a function so no $f^{-1}(x)$)

$x \mid x \geq 0$
$y \mid y \in R$

c) $f(x) = x^2 - 25$ $x \in R$
   $y \geq 25$

$x = y^2 - 25$

$x + 25 = y^2$

$y = \pm\sqrt{x+25}$

$x \mid x \geq 25$
$y \mid y \in R$

d) $f(x) = \sqrt{16 - x^2}$ $x \mid -4 \leq x \leq 4$
   $y \mid 0 \leq y \leq 4$

$x = \sqrt{16 - y^2}$

$x^2 = 16 - y^2$

$y^2 = 16 - x^2$

$y = \pm\sqrt{16 - x^2}$

$x \mid 0 \leq x \leq 4$
$y \mid -4 \leq y \leq 4$

8. In each of the following
   i) sketch the graph of $y = f(x)$ on the grid provided
   ii) determine $f^{-1}(x)$ for the function with restricted domain
   iii) sketch the graph of $y = f^{-1}(x)$ on the grid provided

→ for inverse to be a function must only graph ½ of original $(x \geq 0)$

a) $f(x) = x^2$ $x \geq 0$

$x = y^2$

$y = \sqrt{x}$

$y = \sqrt{x}$

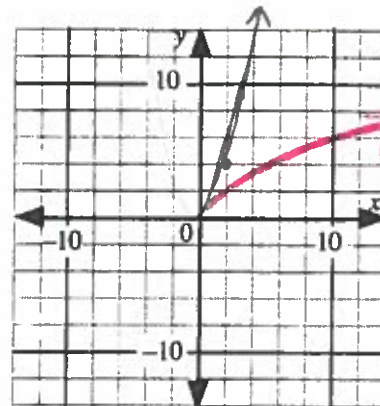note: to make inverse must graph either $y = \sqrt{x}$ $f^{-1}(x) = \sqrt{x}$
   or   or
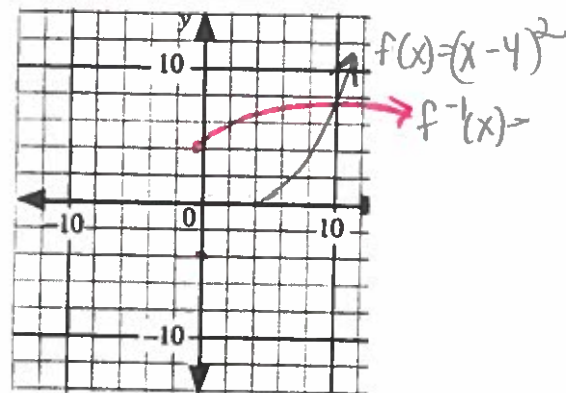$y = -\sqrt{x}$ $f^{-1}(x) = -\sqrt{x}$

b) $f(x) = (x - 4)^2, \ x \geq 4$

$$x = (y-4)^2$$

$$\sqrt{x} = y - 4$$

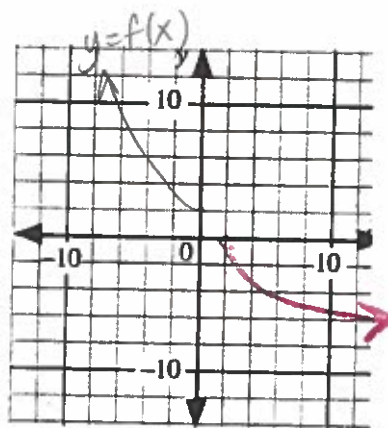$$y = \sqrt{x} + 4$$

$$f^{-1}(x) = \sqrt{x} + 4$$

c) $f(x) = x^2 + 2, \ x \leq 0$

$$x = y^2 + 2$$

$$y^2 = x - 2$$

$$y = \overline{\sqrt{x - 2}}$$

9. Kaleb incorrectly determined the inverse of $y = 4 - \sqrt{-x}$ to be $y = -(x - 4)^2$ and used the graphing calculator to obtain a parabola. Explain why the graph of the correct inverse is not a complete parabola.

10. Functions f and g are defined as $f(x) = 2x + 6$ and $g(x) = 3x$.

   a) Determine $f^{-1}(x)$ and $g^{-1}(x)$.

$x = 2y + 6$ $\qquad\qquad$ $x = 3y$

$\frac{2y}{2} = \frac{x - 6}{2}$ $\qquad\qquad$ $y = \frac{1}{3}x$

$f^{-1}(x) = \frac{1}{2}x - 3$ $\qquad\qquad$ $g^{-1}(x) = \frac{1}{3}x$

   b) Find expressions for

     i) $(f^{-1} \circ g^{-1})(x)$ $\qquad\qquad$ **ii)** $(g^{-1} \circ f^{-1})(x)$

$(f^{-1} \circ g^{-1})(x) = \frac{1}{2}(\frac{1}{3}x) - 3$ $\qquad$ $= \frac{1}{3}(\frac{1}{2}x - 3)$

$\qquad\qquad = \frac{1}{6}x - 3$ $\quad$ same $\qquad\qquad = \frac{1}{6}x - 1$

same

     iii) $(f \circ g)^{-1}(x)$ $\qquad\qquad$ **iv)** $(g \circ f)^{-1}(x)$

replace g into f + find inverse

$\qquad\qquad\qquad = 3(2x + 6)$

$= 2(3x) + 6$ $\qquad\qquad g \circ f = 6x + 18$

$f \circ g = 6x + 6$ $\qquad\qquad x = 6y + 18$

$x = 6y + 6$ $\qquad\qquad\qquad 6y = x - 18$

$\frac{6y}{6} = \frac{x}{6} - \frac{6}{6}$ $\qquad\qquad y = \frac{1}{6}x - 3$

$(f \circ g)^{-1}(x) = \frac{1}{6}x - 1$ $\qquad (g \circ f)^{-1}(x) = \frac{1}{6} - 3$

   c) Compare the answers in b). What do you notice?

$(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

$(g \circ f)^{-1}(x) = (f^{-1} \circ g^{-1})(x)$

Multiple Choice

11. Given that $f(x) = 1 - 2x, x \in R$, then $f^{-1}(x)$ is

A. $-\dfrac{x}{2} - 1$

B. $\dfrac{x}{2} - 1$

C. $\dfrac{1-x}{2}$

D. $\dfrac{x-1}{2}$

$X = 1 - 2y$

$\dfrac{2y}{2} = \dfrac{1}{2} - \dfrac{x}{2}$

$y = \dfrac{1-x}{2}$

12. Given that $f(x) = (x - 2)^2, x \in R$, then the inverse of $f(x)$ is completely defined by

A. $\sqrt{x+2}$

B. $\sqrt{x} + 2$

C. $-\sqrt{x} + 2$

D. none of the above

$X = (y - 2)^2$

$\pm\sqrt{x} = y - 2$

$y = 2 \pm \sqrt{x}$

13. Given that $f(x) = 2x$ and $g(x) = 3 - 5x$, then $(g \circ f)^{-1}(x)$ equals

A. $\dfrac{3}{11}$

B. $\dfrac{6}{11}$

C. $\dfrac{1}{10}(3 - x)$

D. $\dfrac{1}{10}(6 - x)$

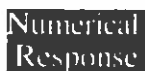$= 3 - 5(2x)$

$gof = 3 - 10x$

$x = 3 - 10y$

$10y = \dfrac{3}{10} - \dfrac{x}{10}$

$(g \circ f)^{-1}(x) = \dfrac{3}{10} - \dfrac{1}{10}x$

$= \dfrac{1}{10}(3 - x)$

$= \dfrac{3}{10} - \dfrac{x}{10}$

Numerical Response

14. The graph of $y = P(x)$ passes through the points $(14, 2)$, $(2, 15)$, and $\left(\dfrac{1}{2}, 10\right)$.

The value of $f^{-1}(2)$ is _____ .

(Record your answer in the numerical response box from left to right.)

| 1 | 4 | | |