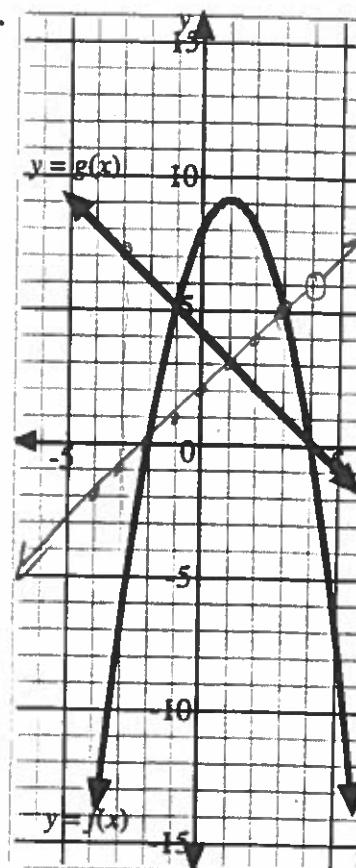


Assignment

1. Two functions $f(x)$ and $g(x)$ are defined for all real numbers. The graphs of the functions are shown on the grid.

a) Complete the table below.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
-4	-16	8	$-16/8 = -2$
-3	-7	1	$-\frac{7}{1} = -7$
-2	0	6	$\frac{0}{6} = 0$
-1	5	5	$\frac{5}{5} = 1$
0	8	4	$\frac{8}{4} = 2$
1	9	3	$\frac{9}{3} = 3$
2	8	2	$\frac{8}{2} = 4$
3	5	1	$\frac{5}{1} = 5$
4	0	0	% undefined
5	-7	-1	$-\frac{7}{-1} = 7$



- b) Explain why the domain of the function $\left(\frac{f}{g}\right)(x)$ is not $x \in R$.

$\left(\frac{f}{g}\right)(x)$ is undefined when $g(x) = 0$ because you can't divide by 0; the domain is $x | x \neq 4, x \in R$.

- c) Sketch the graph of $y = \left(\frac{f}{g}\right)(x)$ showing the domain restriction by drawing an open circle on the graph.

- d) The functions f and g above are $f(x) = 8 + 2x - x^2$ and $g(x) = 4 - x$. Write and simplify an expression for the function $\left(\frac{f}{g}\right)(x)$, including the domain restriction.

$$\frac{8+2x-x^2}{4-x} = \frac{(2+x)(4-x)}{(4-x)} = 2+x, x \neq 4$$

- e) Evaluate $\left(\frac{f}{g}\right)(12)$.

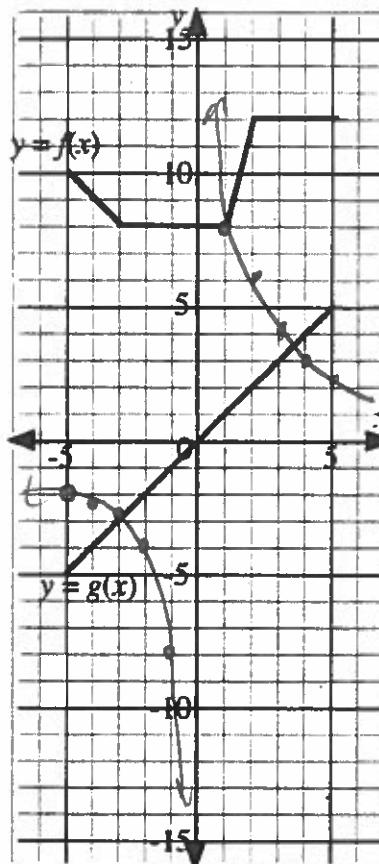
$$\left(\frac{f}{g}\right)(x) = 2+x \\ = 2+12 = \underline{\underline{14}}$$

2. Two functions $f(x)$ and $g(x)$ are defined for all real numbers. The graphs of the functions are shown on the grid.

- a) Complete the table below. Plot the points on the graph

of $y = \left(\frac{f}{g}\right)(x)$, but do not connect the points at this time.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
-5	10	-5	$10/-5 = -2$
-4	9	-4	$9/-4 = -2.25$
-3	8	-3	$8/-3 = -2.66$
-2	8	-2	$8/-2 = -4$
-1	8	-1	$8/-1 = -8$
0	8	0	8/0 undefined
1	8	1	$8/1 = 8$
2	12	2	$12/2 = 6$
3	12	3	$12/3 = 4$
4	12	4	$12/4 = 3$
5	12	5	$12/5 = 2.4$



- b) State the domain of the function $\left(\frac{f}{g}\right)(x)$.

$$x : x \neq 0, x \in \mathbb{R}$$

- c) To investigate the behaviour of the function $\left(\frac{f}{g}\right)(x)$ near the domain restriction, complete the table to the right and plot the points on the grid.

- d) Connect all the points on the grid and complete the graph of $y = \left(\frac{f}{g}\right)(x)$.

x	$f(x)$	$g(x)$	$\left(\frac{f}{g}\right)(x)$
-0.5	8	-0.5	-16
-0.25	8	-0.25	-32
0.25	8	0.25	32
0.5	8	0.5	16

 In this example, the y -axis is a vertical asymptote on the graph of $y = \left(\frac{f}{g}\right)(x)$.

This type of asymptotic behaviour on the graph of a function will be investigated further in the unit on Rational Functions.

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3. In each case, write and simplify (where possible) an expression for $\left(\frac{f}{g}\right)(x)$.

Include any domain restrictions.

a) $f(x) = x + 6, g(x) = x + 3$

$$= \frac{x+6}{x+3}, x \neq -3$$

b) $f(x) = x^2 - x - 20, g(x) = x - 5$

$$\frac{x^2-x-20}{x-5} = \frac{(x-5)(x+4)}{(x-5)} = x+4, x \neq 5$$

c) $f(x) = x + 4, g(x) = x^2 + x - 12$

$$\begin{aligned} &= \frac{x+4}{x^2+x-12} = \frac{1}{x-3}, x \neq -4 \\ &= \frac{x+4}{(x-3)(x+4)} \end{aligned}$$

d) $f(x) = 2x^2 - 7x - 15, g(x) = x - 5$

$$\begin{aligned} &= \frac{2x^2-7x-15}{x-5} = \frac{x+3}{x-5} = \frac{(2x+3)(x-5)}{x-5} \\ &= 2x+3, x \neq 5 \end{aligned}$$

4. Given $f(x) = 5x - 10$ and $g(x) = x - 2$, determine the following functions in simplest form and state any restrictions on x .

a) $(f+g)(x)$

$$= (5x-10) + (x-2)$$

$$= 6x-12$$

$$= 6(x-2)$$

c) $(fg)(x)$

$$= (5x-10)(x-2)$$

$$= 5x^2 - 20x - 20$$

$$= 5(x^2 - 4x - 4)$$

$$= 5(x-2)(x-2)$$

$$= 5(x-2)^2$$

b) $(f-g)(x)$

$$= (5x-10) - (x-2)$$

$$= 4x + 8$$

$$= 4(x+2)$$

d) $\left(\frac{f}{g}\right)(x)$

$$= \frac{5x-10}{x-2}$$

$$= \frac{5(x-2)}{x-2}$$

$$= 5, x \neq 2$$

5. Given $f(x) = x^2 - 9$ and $g(x) = x + 3$, determine the following functions in simplest form and state the restrictions on the variable.

a) $(f+g)(x)$

$$\begin{aligned} &= (x^2 - 9) + (x + 3) \\ &= x^2 + 1x - 6 \end{aligned}$$

or
 $= (x+3)(x-2)$

c) $(fg)(x)$

$$\begin{aligned} &= (x^2 - 9)(x + 3) \\ &= x^3 + 3x^2 - 9x - 27 \end{aligned}$$

b) $(f-g)(x)$

$$\begin{aligned} &= x^2 - 9 - (x + 3) \\ &= x^2 - 1x - 12 \text{ or } (x-4)(x+3) \end{aligned}$$

d) $\left(\frac{f}{g}\right)(x) = \frac{x^2 - 9}{x+3} = \frac{(x-3)(x+3)}{x+3}$
 $= x-3, x \neq -3$

6. Consider the functions $f(x) = 6x^2 + 5x - 6$, and $g(x) = 6x^2 - 13x + 6$

- a) State the domains of f and g .

$$x | x \in \mathbb{R}$$

- b) Write an expression in simplest form for $\left(\frac{f}{g}\right)(x)$. State the domain.

$$\begin{aligned} \frac{6x^2 + 5x - 6}{6x^2 - 13x + 6} &= \frac{(3x-2)(2x+3)}{(3x-2)(2x-3)} \\ &= \frac{2x+3}{2x-3}, x \neq \frac{2}{3}, \frac{3}{2} \end{aligned}$$

$$\begin{array}{r} 6x^2 + 5x - 6 \\ -3x + 5 \\ \hline 6x^2 - 13x + 6 \\ +9x - 9 \\ \hline 3x(2x+3) - 2(2x+3) \end{array}$$

- c) Show two different ways to evaluate $\left(\frac{f}{g}\right)(4)$.

$$\begin{aligned} \textcircled{1} \quad &\frac{6x^2 + 5x - 6}{6x^2 - 13x + 6} \\ &= \frac{6(4)^2 + 5(4) - 6}{6(4)^2 - 13(4) + 6} \\ &= 11/5 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad &\frac{2x+3}{2x-3} \\ &= \frac{2(4)+3}{2(4)-3} \\ &= 11/5 \end{aligned}$$

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7. In each case, find two functions f and g such that

a) $(f+g)(x) = x^2 + 3x$
 $f(x) = x^2 \quad g(x) = 3x$

$f(x) = x^2 + 1x \quad g(x) = 2x$

c) $(fg)(x) = x^2 + 3x$

$f(x) = x$

$g(x) = x + 3$

b) $(f-g)(x) = x^2 + 3x$
 $f(x) = 2x^2 + 6x$

$g(x) = x^2 + 3x$

d) $\left(\frac{f}{g}\right)(x) = x^2 + 3x$

$f(x) = x^3 + 3x^2$

$g(x) = x$

8. Given $f(x) = \frac{x}{x-3}$ and $g(x) = \frac{2x}{x+1}$, determine the following functions in simplest form and state any restrictions on x .

a) $(f+g)(x)$

$$= \frac{x(x+1)}{x-3} + \frac{2x(x-3)}{x+1}$$

$$\frac{x^2 + 1x + 2x^2 - 6x}{(x-3)(x+1)}$$

$$= \frac{3x^2 - 5x}{(x-3)(x+1)} \text{ or } \frac{x(3x-5)}{(x-3)(x+1)}$$

$x \neq -1, 3$

b) $(f-g)(x)$ careful

$$\frac{x(x+1)}{x-3} - \frac{2x(x-3)}{x+1}$$

$$= \frac{x^2 + 1x - 2x^2 + 6x}{(x-3)(x+1)}$$

$$= -\frac{x^2 + 7x}{(x-3)(x+1)} \text{ or } \frac{-x(-1-x)}{(x-3)(x+1)}$$

$$x \neq -1, 3$$

c) $(fg)(x)$

$$\left(\frac{x}{x-3}\right) \left(\frac{2x}{x+1}\right)$$

$$= \frac{2x^2}{(x-3)(x+1)}, \quad x \neq -1, 3$$

d) $\left(\frac{f}{g}\right)(x)$

$$= \frac{x}{x-3} \div \frac{2x}{x+1}$$

$$= \frac{x}{x-3} \cdot \frac{x+1}{2x}$$

$$= \frac{x+1}{2(x-3)}, \quad x \neq -1, 0, 3$$

answers will
vary

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9. Given $f(x) = x + 1$ and $g(x) = x^2 - 1$, determine the following functions in simplest form and state any restrictions on x .

a) $3(f-g)(x) = 3 \left[(x+1) - (x^2 - 1) \right]$
 $= 3 \left(-x^2 + x + 2 \right) = -3x^2 + 3x + 6$

b) $(fg)(x) - g(x)$
 $= (x+1)(x+1) - (x^2 - 1) = x^2 + 2x + 1 - (x^2 - 1) = 2x + 2$

c) $\left(\frac{g}{f}\right)(x) = \frac{x^2 - 1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x-1, x \neq -1$

Multiple Choice

10. Consider the functions $f(x) = \frac{x-4}{x+2}$ and $g(x) = \frac{x-3}{x-1}$. Which of the following are restrictions for $\left(\frac{f}{g}\right)(x)$? $x \neq -2, x \neq 1$

- A. -2 and 1 only
 B. -2, 1, and 3 only
 C. -2, 1, and 4 only
 D. -2, 1, 3, and 4

$$\frac{x-4}{x+2} \div \frac{x-3}{x-1} \rightsquigarrow \frac{x-4}{x+2} \cdot \frac{x-1}{x-3}$$

$$x \neq -2, 3, 1$$

Numerical Response

11. If $f(x) = 2x^2$ and $g(x) = \frac{x-4}{2x}$, determine the values of:

1. $(fg)(3)$

2. $(f-g)(4)$

3. $\left(\frac{f}{g}\right)(2)$

4. $(f+g)(1)$

Rearrange the four answers in increasing order. Write the question number corresponding to the smallest answer in the first box, the question number corresponding to the second smallest answer in the second box, etc.

(Record your answer in the numerical response box from left to right.)

3	1	4	2
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$$\begin{aligned}
 1. &= 2x^2 \left(\frac{x-4}{2x} \right) & 2. &= 2x^2 - \left(\frac{x-4}{2x} \right) & 3. &= \frac{2x^2}{\frac{(x-4)}{2x}} & 4. &= 2x^2 + \frac{x-4}{2x} \\
 &= 2(3)^2 \left[\frac{3-4}{2(3)} \right] & &= 2(4)^2 - \left(\frac{4-4}{2(4)} \right) & &= 2(1)^2 + \frac{1-4}{2(1)} & &= 2 + (-1.5) \\
 &= -3 & &= 32 & &= \frac{8}{-0.5} & &= 0.5 \\
 & \textcircled{2} & & \textcircled{4} & & \textcircled{1} & & \textcircled{3}
 \end{aligned}$$