

Assignment

1. Consider the functions $f(x) = 3x + 7$ and $g(x) = 2x - 4$, defined for all real numbers.

a) Determine the following functions in simplest form and state the domain and range of each function.

i) $(f+g)(x)$
 $= (3x+7) + (2x-4)$
 $= 5x+3$

ii) $(f-g)(x)$
 $= 3x+7 - (2x-4)$
 $= 3x+7-2x+4$
 $= x+11$

iii) $(g-f)(x)$
 $= 2x-4 - (3x+7)$
 $= 2x-4-3x-7$
 $= -x-11$
 $= -x-11$
 $= -11$

b) Evaluate

i) $(f+g)(5)$
 $= 5(5)+3$
 $= 28$

ii) $(f-g)(-3)$
 $= (-3)+11$
 $= 8$

iii) $(g-f)(0)$
 $= 0-11$
 $= -11$

2. If $P(x) = x^2 - 7x + 3$ and $Q(x) = 2x + 9$, evaluate $(P-Q)(-3)$ in two different ways.

$$(P-Q)(x) = x^2 - 7x + 3 - (2x+9)$$

$$= x^2 - 9x - 6$$

$$(P-Q)(-3) = (-3)^2 - 9(-3) - 6$$

$$= 30$$

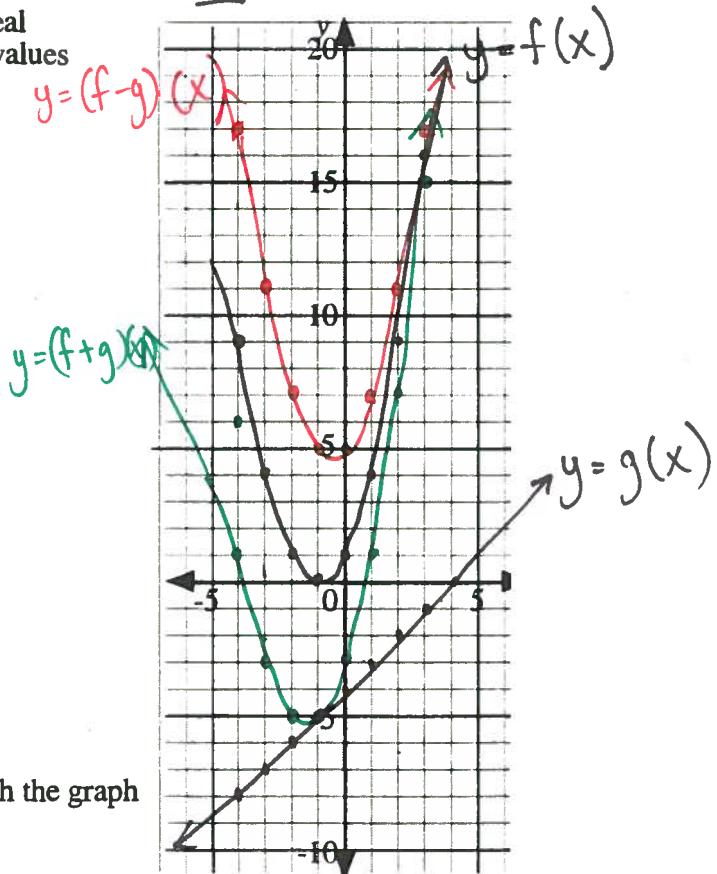
$$P(-3) - Q(-3)$$

$$= [(-3)^2 - 7(-3) + 3] - [2(-3) + 9]$$

$$= \underline{\underline{30}}$$

3. Two functions $f(x)$ and $g(x)$ are defined for all real numbers. The table of values below shows the values of f and g for certain values of x .

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$
-4	9	-8	1	17
-3	4	-7	-3	11
-2	1	-6	-5	7
-1	0	-5	-5	5
0	1	-4	-3	5
1	4	-3	1	7
2	9	-2	7	11
3	16	-1	15	17
4	25	0	25	25



- a) Plot the points $(x, f(x))$ on the grid and sketch the graph of $y = f(x)$ for $x \in R$. ✓
- b) Plot the points $(x, g(x))$ on the grid and sketch the graph of $y = g(x)$ for $x \in R$. ✓
- c) Complete the tables above for $(f+g)(x)$ and $(f-g)(x)$ and sketch the graphs of $y = (f+g)(x)$ and $y = (f-g)(x)$ for $x \in R$.

- d) The functions f and g on the previous page are $f(x) = x^2 + 2x + 1$ and $g(x) = x - 4$.

Write expressions for the functions $(f + g)(x)$ and $(f - g)(x)$.

$$(f+g)(x) = x^2 + 2x + 1 + (x-4) \quad (f-g)(x) = x^2 + 2x + 1 - (x-4)$$

$$= x^2 + 3x - 3 \quad = x^2 + x + 5$$

- e) Use a graphing calculator to graph the functions $y = (f + g)(x)$ and $y = (f - g)(x)$ from part d) and compare these graphs with the graphs from part c).

- f) State the domains of the functions $(f + g)(x)$ and $(f - g)(x)$.

$$x \in \mathbb{R}.$$

- g) Determine the values of $(f + g)(8)$ and $y = (f - g)(-12)$.

$$(f+g)(8) = 8^2 + 3(8) - 3 \quad (f-g)(-12) = (-12)^2 + (-12) + 5$$

$$= 85 \quad = 137$$

- h) In this example, the sum and difference of a linear function and a quadratic function are quadratic functions. Can you find a linear function and a quadratic function whose sum or difference is not a quadratic function?

No.

4. The graphs of two quadratic functions, $y = f(x)$ and $y = g(x)$, $x \in \mathbb{R}$, are shown on the grid.

- a) Use these graphs to sketch the graphs of $y = (f + g)(x)$ and $y = (f - g)(x)$ for $x \in \mathbb{R}$.

- b) The functions f and g above are $f(x) = 4 - x$ and $g(x) = -x^2 + 3x - 3$. Write expressions for the functions $(f + g)(x)$ and $(f - g)(x)$.

$$(f+g)(x) = 4 - x + (-x^2 + 3x - 3)$$

$$= -x^2 + 2x + 1$$

$$(f-g)(x) = 4 - x - (-x^2 + 3x - 3)$$

$$= x^2 - 4x + 7.$$

- c) Use a graphing calculator to graph the functions $y = (f + g)(x)$ and $y = (f - g)(x)$ from part b) and compare these graphs with the graphs from part a).

- d) State the domains of the functions $(f + g)(x)$ and $(f - g)(x)$.

$$x \in \mathbb{R}.$$

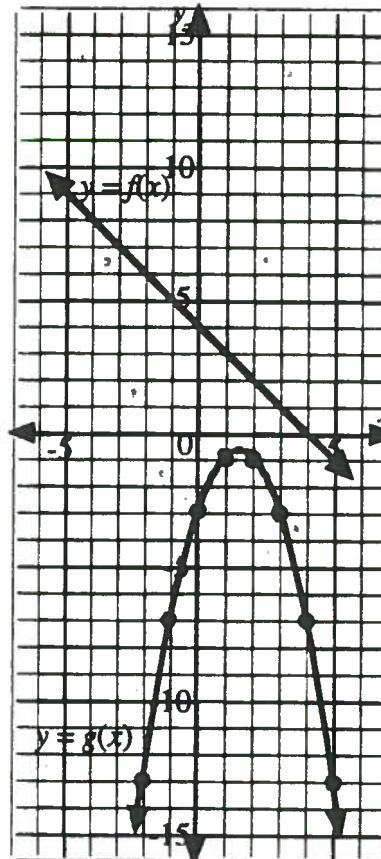
- e) Evaluate. i) $(f + g)(-10)$ ii) $(f - g)(\sqrt{3})$

$$(f+g)(-10) = -(-10)^2 + 2(-10) + 1$$

$$= -119$$

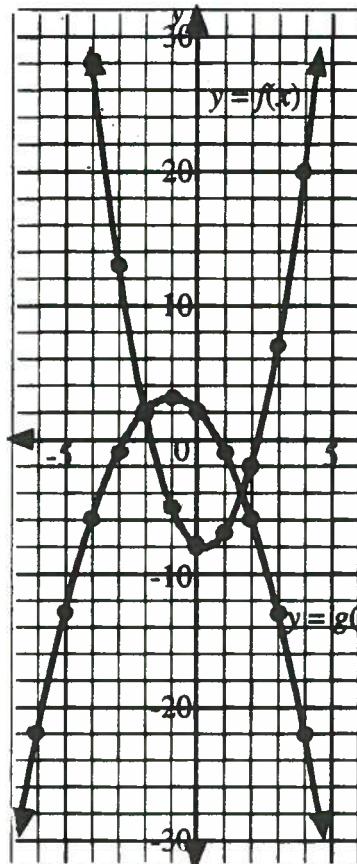
$$(f-g)(\sqrt{3}) = (\sqrt{3})^2 - 4(\sqrt{3}) + 7$$

$$= 10 - 4\sqrt{3}$$



14 Functions and Relations Lesson #2: Operations with Functions - Part One

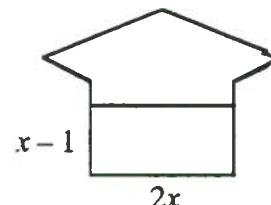
5. The graphs of two quadratic functions, $y = f(x)$ and $y = g(x)$, $x \in R$, are shown on the grid.
- Use these graphs to sketch the graphs of $y = (f - g)(x)$ and $y = (g - f)(x)$ for $x \in R$.
 - How do the graphs of $y = (f - g)(x)$ and $y = (g - f)(x)$ relate to each other?
 - The functions f and g above are $f(x) = 2x^2 - x - 8$ and $g(x) = -x^2 - 2x + 2$. Write expressions for the functions $(f - g)(x)$ and $(g - f)(x)$.
 - Use a graphing calculator to graph the functions $y = (f - g)(x)$ and $y = (g - f)(x)$ from part c) and compare these graphs with the graphs from part a).
 - Evaluate. i) $(f - g)\left(\frac{1}{2}\right)$ ii) $(g - f)(2\sqrt{2})$



6. The figure shown has an area $A(x) = 2x^2 + 8x - 3 \text{ cm}^2$.

- a) Write an expression for the area, $B(x)$, of the bottom rectangular part of the figure.

$$\begin{aligned} B(x) &= 2x(x-1) \\ &= 2x^2 - 2x \text{ cm}^2 \end{aligned}$$



- b) Find an expression in simplest form for the area, $T(x)$, of the top part of the figure.

$$\begin{aligned} T(x) &= A(x) - B(x) \\ &= 2x^2 + 8x - 3 - (2x^2 - 2x) \\ &= 10x - 3 \text{ cm}^2 \end{aligned}$$

- c) If the area of the top is 9 cm^2 , determine the value of x .

$$10x - 3 = 9$$

$$10x = 12$$

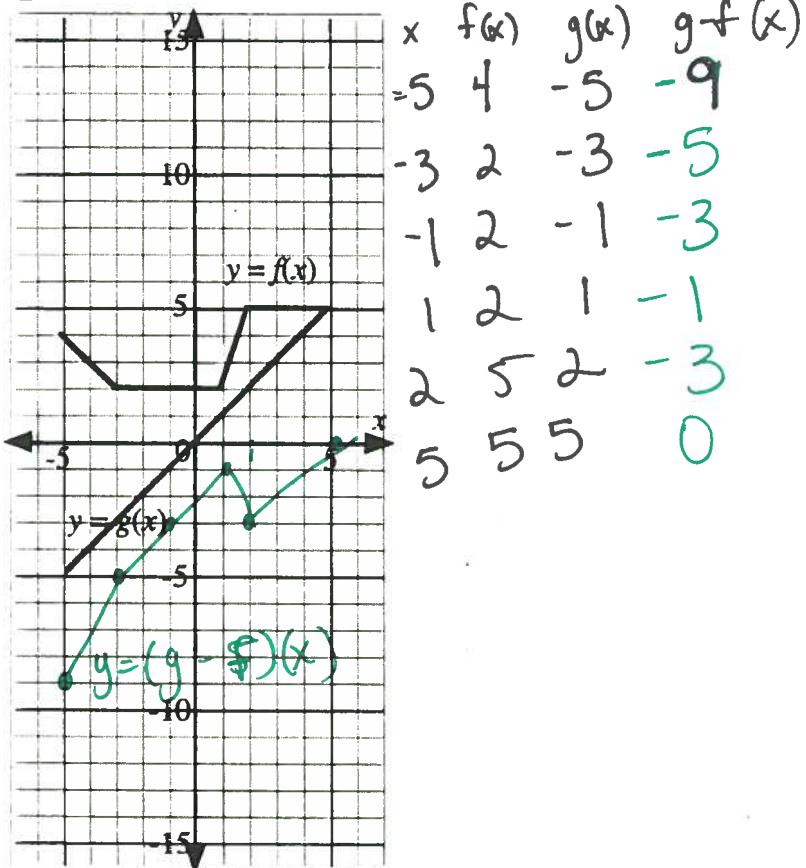
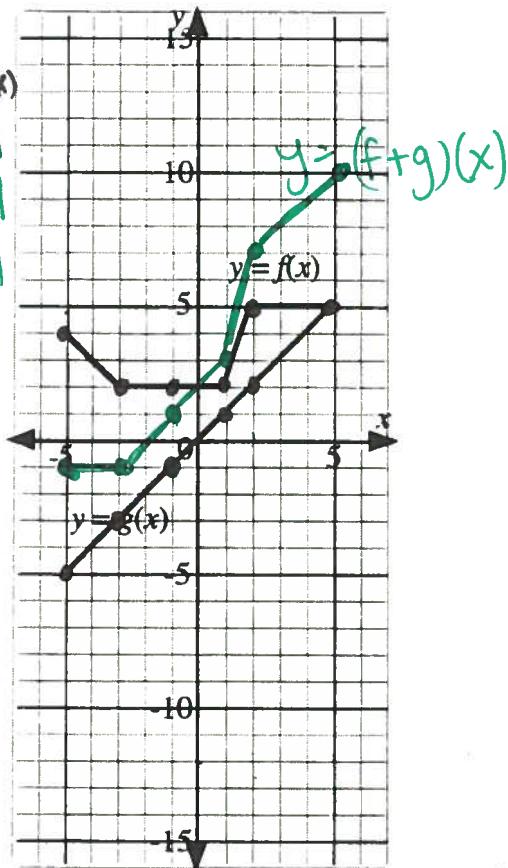
$$x = \frac{6}{5}$$

7. Partial graphs of functions f and g are shown on the grids.

In each case, sketch the graph of the indicated function.

a) $(f + g)(x)$

b) $(g - f)(x)$



Multiple Choice

8. The graph of $y = f(x)$ includes the points $(2, 5)$, $(4, 15)$, $(7, 30)$.
The graph of $y = g(x)$ includes the points $(2, 6)$, $(3, 16)$, $(4, 30)$.

If f and g are defined for all real values of x , then which of the following points must lie on the graph of $y = (f - g)(x)$?

- A. $(0, -1)$ B. $(0, 1)$
 C. $(2, -1)$ D. $(3, 30)$

$$(2, 5) \quad 15 - 6 = -1$$

$$(2, 6) \quad \underline{\quad}$$

$$(2, -1) \quad \underline{\quad}$$

Numerical Response

9. If $f(x) = \sqrt{3x} - 2$ and $g(x) = 5\sqrt{3x}$, then $2(g - f)(3)$, to the nearest tenth, is _____.
(Record your answer in the numerical response box from left to right.)

$$\begin{aligned} 2(g - f)(x) &= 2(5\sqrt{3x} - (\sqrt{3x} - 2)) \\ &= 2(4\sqrt{3x} + 2) \\ &= 8\sqrt{3x} + 4 \end{aligned}$$

$$\begin{aligned} 2(g - f)(3) &= 8\sqrt{3 \cdot 3} + 4 \\ &= 8\sqrt{9} + 4 \\ &= 8(3) + 4 \\ &= 28 \end{aligned}$$

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